

Big – Oh Notation:

Definition: $T(n) = O(f(n))$ if and only if there exist constance $c, n_0 > 0$ such that $T(n) \leq cf(n)$ for all $n \geq n_0$.

Claim: if $T(n) = a_k n^k + \dots + a_1 n + a_0$, then $T(n) = O(n^k)$

Proof:

$$\begin{aligned} T(n) &= a_k n^k + \dots + a_1 n + a_0 \\ &\leq |a_k| n^k + \dots + |a_1| n + a_0 \\ &\leq |a_k| n^k + \dots + |a_1| n^k + |a_0| n^k \\ &= (|a_k| + \dots + |a_1| + |a_0|) n^k \\ &= c n^k \end{aligned}$$

$$c = (|a_k| + \dots + |a_1| + |a_0|)$$

$$n_0 = 1$$

therefore $T(n) = O(n^k)$

Big – Omega Notation:

Definition: $T(n) = O(f(n))$ if and only if there exist constance $c, n_0 > 0$ such that $T(n) \geq cf(n)$ for all $n \geq n_0$.

Ex1.

```
Void max(int[]a){
    Int max = a[0];
    For(int l =1;i<a.length;i++){
        If(max < a[i]){
            max = a[i];
        }
    }
}
```

$O(n)$

Ex2 .

```
Int sum(int a[]){
    Int sum = 0;
    For(int l =0;i< a.length;i++){
        Sum+= a[i];
    }
    Return sum;
}
```

$O(n)$

Ex3.

```
Int sum(int a[],int index){
    If(index == a.length)
        Return 0;
    Return a[i] + sum(a,index+1);
}
```

$$T(n) = T(n-1) + 1$$

$$T(n-1) = T(n-2) + 1$$

$$T(n-2) = T(n-3) + 1$$

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$$T(1) = 1$$

$$T(n) = T(n-1) + 1$$

$$= T(n-2) + 1 + 1$$

$$= T(n-3) + 1 + 1 + 1$$

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$$= T(1) + 1 + 1 + \dots + 1$$

$$= n$$

$$T(n) = O(n)$$

Ex4

```
Boolean search(int a[],int t){
    for(int i=0; i<a.length; i++){
        If(a[i] == t){
            Return true;
        }
    }
    return false;
}
```

$$O(n)$$

Ex5

```
Boolean search(int a[], int l ,int r,int t){
    If(r > l)
        Return Boolean;
    Int m = l + (r- l)/2;
    If( a[m] == t){return true;}
    If(a[m] < t)
```

```
        Return search(a, m+1, r,t);
    Return search(a, l, m-1,r);
}
```

$$T(n) = T(n/2) + 1$$

$$T(n) = \log n$$