Designing

Not all designs are equally good.

Universal Relation is a very bad idea.

How to decompose relation? When to decompose? Functional dependency used in normalization
Functional dependency

Let’s say
Relation $R$ has attributes (A,B,C)

When A functionally determines B, then each value of A is associated with exactly one value of B. Attribute B is functionally dependent on attribute A is denoted as $A \rightarrow B$

Recall key: if a pair of tuples has same key, they agree on the rest attributes

- FD is another integrity constraint

$sId \rightarrow sName$

<table>
<thead>
<tr>
<th>sid</th>
<th>sName</th>
</tr>
</thead>
<tbody>
<tr>
<td>12345</td>
<td>John</td>
</tr>
<tr>
<td>56789</td>
<td>Mary</td>
</tr>
<tr>
<td>54321</td>
<td>John</td>
</tr>
</tbody>
</table>
Functional dependency

R has attributes A,B,C;
A \rightarrow B
- Full functional dependency (not on any proper subset of A)

- Partial functional dependency
A, B \rightarrow C
A \rightarrow C
sId, sAddress \rightarrow sName

- Transitive dependency
A \rightarrow B
B \rightarrow C
C is transitive dependent on A via B
staffId \rightarrow staffName, title, department
department \rightarrow location
Identify functional dependency

- Documentation

- Understand the meaning of relationships on attributes

- Sample data, FD true for all time (not a specific content to conclude FD)
Inference Rules for FD

- identify set of FD called $X$, such that the complete set of FD is implied by the $X$, called closure of $X$ ($X^+$)

- Armstrong’s axiom specified inference rules are Sound and Complete
  - Reflexivity:
    If $B$ is a subset of $A$, then $A \rightarrow B$
  - Augmentation:
    If $A \rightarrow B$, then $A, C \rightarrow B, C$
  - Transitivity:
    if $A \rightarrow B$ and $B \rightarrow C$, then $A \rightarrow C$
Inference Rules for FD

Further rules can be derived from the previous one. A, B, C, D are set of attributes of R

- **Self-determination:**
  \[ A \rightarrow A \]

- **Decomposition:**
  If \( A \rightarrow B, C \), then \( A \rightarrow B \) and \( A \rightarrow C \)

- **Union:**
  If \( A \rightarrow B \) and \( A \rightarrow C \), then \( A \rightarrow B, C \)

- **Composition:**
  If \( A \rightarrow B \) and \( C \rightarrow D \), then \( A, C \rightarrow B, D \)
Department Staff FD

staffId → staffName, title, department
department → location

Which of the following is not in the closure of department staff FD?

staffId → staffName

staffId → location

department, staffName → location, staffName

department, staffName → location, title
department, staffName → location
department, staffName → title
department, staffName +
department, staffName, location
Sample

Consider relational schema R = ABCDE, with FD F as following,
1. D \rightarrow E
2. AB \rightarrow C
3. D \rightarrow A
determine if the following dependency belongs to F+
BD \rightarrow C
- the dependency is in F+ if C is in BD+
- compute the closure of F is expensive, instead, compute the closure of attribute sets on the left-hand side

<table>
<thead>
<tr>
<th>dependency</th>
<th>attributes</th>
<th>BD^+</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>E</td>
<td>BD</td>
</tr>
<tr>
<td>3</td>
<td>A</td>
<td>BDE</td>
</tr>
<tr>
<td>2</td>
<td>C</td>
<td>ABCDE</td>
</tr>
</tbody>
</table>
Sample 2

Consider same relational schema and FD
1. D \rightarrow E
2. AB \rightarrow C
3. D \rightarrow A
determine if the following dependency belongs to F+
BE \rightarrow C
- compute BE+
dependency attributes BE+ BE
Equivalence

- Two sets of FD are said to be equivalence, if the closure of two sets are the same

How to check?
Two FD sets $F$ and $G$ is equivalent, if $F$ is covered by $G$ and $G$ is covered by $F$
Covered by

- One set of FD is covered by another set of FD, if every FD in the first one can be inferred from the second one.
Equivalent Sample

- Check whether following two FD set are equivalent

F
A → B
B → C
AB → D

G
A → B
B → C
A → D

Check closure of AB under G
Check closure of A under F
Minimal set

Minimal set of FD if,
- every dependency on the right-hand side is a single attribute

- can’t replace left-hand side attributes with a proper set of itself and still have the equivalent set of dependencies

- can’t remove any dependency and still have the equivalent set of dependencies
Minimal Cover

- minimal set that’s equivalent to the given FD
- there maybe serveral minimal covers for a set of functional dependencies
How to find minimal Cover

- Union
- Simplify LHS
- Simplify RHS
Exercise

Is following dependency set a minimal cover?
1. $A \rightarrow B$
2. $B \rightarrow C$
3. $AC \rightarrow D$

Check if $A \rightarrow D$

transitivity:
$A \rightarrow C$

self-determination:
$A \rightarrow A$

union:
$A \rightarrow AC$

transitivity:
$A \rightarrow D$
Exercise

Is following dependency set a minimal cover?
1. A → B
2. B → C
3. BD → E
4. CD → E
Minimum Cover

Implies Lossless-join, dependency preserving decomposition.

Given a set of FD set F, find another FD set G that is smaller and equivalent (if F is covered by (can be inferred from) G and G is covered by F), steps:

1. Union
2. Simplify LHS (remove redundant attribute)
3. Simplify RHS (remove redundant dependency)
Exercise

How to get minimum cover?
1. A → B
2. AB → C

Simplify LHS
If AB is the determinante and B is in closure of A⁺, then B is the redundant attribute
2 becomes A → C
Exercise

How to get minimum cover?
1. A → B
2. B → C
3. A → C

Union:
A → BC
B → C

Simplify RHS
If dependent C of A maybe determine from another dependent B of A, that C is in closure of B⁺, then A → C is a redundant dependency

Is B → C redundant?
Exercise

Which one of following we may remove?
1. A → B
2. B → C
3. BD → E
4. CD → E

Apply Augmentation to 2:
BD → CD

Apply Transitivity to above use 4
E is in the closure of BD⁺

(CD is in close of BD⁺, thus, BD determines what CD determines is redundant)

3 is redundant

BD → CDE (simplify RHS) -- remove
CD → E
Check Equivalence

if FD set F is equivalent to FD set G, then F can be inferred from G and G can be inferred from F

\[ F \equiv G \]

\[ GF \]

\[ FG \]

\[ G \]
Exercise

Given FD set F:
1. A $\rightarrow$ BI
2. B $\rightarrow$ CE
3. D $\rightarrow$ B

FD set G:
1. A $\rightarrow$ BC
2. B $\rightarrow$ CE
3. D $\rightarrow$ B
4. D $\rightarrow$ E
5. AC $\rightarrow$ I

Check if F can be inferred from G
See BI is in the closure of A$^+$ under G
2 and 3 is in G
Exercise

Given FD set F:
1. $A \rightarrow BI$
2. $B \rightarrow CE$
3. $D \rightarrow B$

FD set G:
1. $A \rightarrow BC$
2. $B \rightarrow CE$
3. $D \rightarrow B$
4. $D \rightarrow E$
5. $AC \rightarrow I$

Check if G can be inferred from F
See BC is in the closure of $A^+$ under F
2 and 3 is in F
See E is in the closure of $D^+$ under F
See I is in the closure of $AC^+$ under F
Thus, F and G are equivalent