# CS320: Additional midterm review problems and solutions, Winter 2023

I advise you to try to solve these problems before reading their solutions. You learn more by solving on your own and learn most by trying, failing to solve and then reading the solution.

**Problem 1** Let $L$ be the language defined by the regular expression:

$$(\boldsymbol{a} \cup \boldsymbol{b} \cup \boldsymbol{c})((\boldsymbol{ab} \cup \boldsymbol{ac} \cup \boldsymbol{b})^*(\boldsymbol{a} \cup \boldsymbol{b}) \cup (\boldsymbol{aa})^*)$$

**(a)** Write 10 distinct strings that belong to $L$. If such strings do not exist, explain why.

**(b)** Write 10 distinct strings over alphabet $\{a, b, c\}$ that do not belong to $L$. If such strings do not exist, explain why.

**Answer:**

| $\in L$ | $\notin L$ |
|---|---|
| $a$ | $\lambda$ |
| $aa$ | $cc$ |
| $aaa$ | $ccc$ |
| $aaaaa$ | $cccc$ |
| $abb$ | $bc$ |
| $babacbba$ | $ac$ |
| $b$ | $caaa$ |
| $baaaaaa$ | $baaa$ |
| $c$ | $aaaa$ |
| $caaaa$ | $aaaaaa$ |

**Problem 2** Let $L$ be the language defined by the regular expression

$$\boldsymbol{c}^*(\boldsymbol{b} \cup (\boldsymbol{ac}^*))^*$$

**(a)** Write 10 distinct strings that belong to $L$.

**(b)** Write 10 distinct strings over alphabet $\{a, b, c\}$ that do not belong to $L$.

**Answer:** Note that $L$ is the set of strings over $\{a, b\}$ that do not contain $bc$ as a substring.

| $\in L$ | $\notin L$ |
|---|---|
| $c$ | $bc$ |
| $cb$ | $bcc$ |
| $cac$ | $bcabc$ |
| $cbac$ | $cbcac$ |
| $b$ | $bbc$ |
| $ac$ | $abc$ |
| $aaaaccc$ | $aaaabcc$ |
| $bbbacac$ | $bbbabcac$ |
| $cccbbac$ | $cccbcac$ |
| $ccbacccbb$ | $ccbccccbb$ |

**Problem 3** Let $L$ be the language defined by the regular expression

$$(\boldsymbol{b} \cup \boldsymbol{ab} \cup \boldsymbol{aab})^*(\boldsymbol{\lambda} \cup \boldsymbol{a} \cup \boldsymbol{aa})$$

**(a)** Write 10 distinct strings that belong to $L$.

**(b)** Write 10 distinct strings over alphabet $\{a, b\}$ that do not belong to $L$.

**Answer:** Note that $L$ is the set of strings over $\{a, b\}$ that do not contain $aaa$ as a substring.

| $\in L$ | $\notin L$ |
|---|---|
| $b$ | $aaa$ |
| $ba$ | $aaaaa$ |
| $baa$ | $baaa$ |
| $ab$ | $aaab$ |
| $aba$ | $abaaa$ |
| $abaa$ | $aabaaa$ |
| $aab$ | $aaaab$ |
| $aaba$ | $aaaaba$ |
| $aabaa$ | $aabaaaa$ |
| $bbbbbbabaabaa$ | $bbbbbbabaaabaa$ |

**Problem 4** Let $L$ be the set of strings over alphabet $\{a, b, c\}$ that contain as a substring at least one of the strings: $ba$, $bc$.

**(a)** Write 5 distinct strings that belong to $L$.

**(b)** Write 5 distinct strings over alphabet $\{a, b, c\}$ that do not belong to $L$.

**Answer:**

| $\in L$ | $\notin L$ |
|---|---|
| $ba$ | $\lambda$ |
| $bc$ | $a$ |
| $baabc$ | $b$ |
| $bcbbba$ | $c$ |
| $bbababcbcbaaa$ | $ab$ |

**(c)** Write a regular expression that defines $L$.

**Answer:**

$$(\boldsymbol{a} \cup \boldsymbol{b} \cup \boldsymbol{c})^* \boldsymbol{ba} (\boldsymbol{a} \cup \boldsymbol{b} \cup \boldsymbol{c})^* \cup (\boldsymbol{a} \cup \boldsymbol{b} \cup \boldsymbol{c})^* \boldsymbol{bc} (\boldsymbol{a} \cup \boldsymbol{b} \cup \boldsymbol{c})^*$$

**Problem 5** Let $L$ be the set of strings over alphabet $\{0, 1\}$ in which the number of 1s is even.

**(a)** Write 5 distinct strings that belong to $L$. If such strings do not exist, explain why.

**(b)** Write 5 distinct strings over alphabet $\{0, 1\}$ that do not belong to $L$. If such strings do not exist, explain why.

**Answer:**

| $\in L$ | $\notin L$ |
|---|---|
| $\lambda$ | 1 |
| 000 | 01 |
| 011 | 10 |
| 0011 | 001 |
| 110010111 | 01101101 |

**(c)** Write a regular expression that defines $L$. If such expression does not exist, explain why.

**Answer:**

$$(\mathbf{0}^* \mathbf{1} \mathbf{0}^* \mathbf{1} \mathbf{0}^*)^* \cup \mathbf{0}^*$$

**Problem 6** Let $L$ be the set of strings over alphabet $\{a, b, c\}$ that have odd length or odd number of $b$'s. Write a regular expression that defines $L$. If such regular expression does not exist, prove it.

**Answer:**

$$\begin{gathered}(\boldsymbol{a} \cup \boldsymbol{b} \cup \boldsymbol{c})\,(\,(\boldsymbol{a} \cup \boldsymbol{b} \cup \boldsymbol{c})(\boldsymbol{a} \cup \boldsymbol{b} \cup \boldsymbol{c})\,)^* \\ \cup \\ (\boldsymbol{a} \cup \boldsymbol{c})^* \boldsymbol{b}\,(\boldsymbol{a} \cup \boldsymbol{c})^* (\,(\boldsymbol{a} \cup \boldsymbol{c})^* \boldsymbol{b}\,(\boldsymbol{a} \cup \boldsymbol{c})^* \boldsymbol{b}\,(\boldsymbol{a} \cup \boldsymbol{c})^*\,)^*\end{gathered}$$

**Problem 7** Let $L$ be the language generated by the context-free grammar $G = \{V, \Sigma, P, S\}$, where $\Sigma = \{a, b\}$, $V = \{S, A\}$, and $P$ comprises:

$$\begin{aligned} S &\to aA \mid bA \mid \lambda \\ A &\to aS \mid bS \end{aligned}$$

**(a)** Write 10 distinct strings that belong to $L$. If such strings do not exist, explain why.

**(b)** Write 10 distinct strings over alphabet $\{a, b\}$ that do not belong to $L$. If such strings do not exist, explain why.
**Answer:** $L$ is the set of strings of even length over $\Sigma$.

| $\in L$ | $\notin L$ |
|---|---|
| $\lambda$ | $a$ |
| $aa$ | $b$ |
| $bb$ | $aab$ |
| $ba$ | $aba$ |
| $ab$ | $abb$ |
| $abba$ | $bba$ |
| $abbbaa$ | $bbb$ |
| $aaaaaaaa$ | $bab$ |
| $baaaaaaa$ | $baa$ |
| $bbaaaaaa$ | $aaa$ |

**Problem 8** Let $L_1$ be the language defined by the regular expression:

$$\boldsymbol{(a \cup b)^* \, aa \, (a \cup b)^* \, bb \, (a \cup b)^*}$$

Let $L_2$ be the language generated by the context-free grammar $G = \{V, \Sigma, P, S\}$, where $\Sigma = \{a, b\}$, $V = \{S, A, B\}$, and the production set $P$ is:

$$\begin{aligned}&S \rightarrow SS \mid A \mid \lambda\\&A \rightarrow BBB\\&B \rightarrow a \mid b\end{aligned}$$

**(a)** Write 5 distinct strings that belong to $L_1$ and do not belong to $L_2$. If such strings do not exist, explain why.
**Answer:**

$$aabb, aaabb, baabb, aabba, baababb$$

**(b)** Write 5 distinct strings that belong to $L_2$ and do not belong to $L_1$. If such strings do not exist, explain why.
**Answer:**

$$aaa, bbb, aba, bab, aab$$

**(c)** Write 5 distinct strings that belong to $L_1$ and $L_2$. If such strings do not exist, explain why.
**Answer:**

$$aaabbb, baabbb, aaabba, baabba, aababb$$

**(d)** Write 5 distinct strings over alphabet $\{a, b\}$ that do not belong to $L_1$ and do not belong to $L_2$. If such strings do not exist, explain why.
**Answer:**

$$a, b, ab, ba, aa$$

**Problem 9** Let $L$ be the language generated by the context-free grammar $G = \{V, \Sigma, P, S\}$, where $\Sigma = \{a, b, c\}$, $V = \{S, A, B, D\}$, and $P$ comprises:

$$\begin{aligned}&S \rightarrow SS \mid A \mid \lambda\\&A \rightarrow BD\\&B \rightarrow a \mid b\\&D \rightarrow ccc\end{aligned}$$

**(a)** Write 5 distinct strings that belong to $L$.
**(b)** Write 5 distinct strings over alphabet $\{a, b, c\}$ that do not belong to $L$.
**Answer:**

| $\in L$ | $\notin L$ |
|---|---|
| $\lambda$ | $a$ |
| $accc$ | $b$ |
| $bccc$ | $c$ |
| $acccaccc$ | $ac$ |
| $bcccaccc$ | $aa$ |

**(c)** Write a regular expression that defines $L$. If such regular expression does not exist, prove it.
**Answer:**

$$((\boldsymbol{a} \cup \boldsymbol{b})\, \boldsymbol{ccc})^*$$

**Problem 10** Let $L$ be the language generated by the context-free grammar $G = \{V, \Sigma, P, S\}$, where $\Sigma = \{a, b, c, d\}$ is the set of terminals; $V = \{S, A, B, T\}$ is the set of variables; $S$ is the start symbol; and the set of productions $P$ comprises:

$$\begin{aligned} &S \to TA \\ &T \to TT \mid Bd \mid \lambda \\ &B \to BB \mid bc \mid \lambda \\ &A \to aa \end{aligned}$$

Write a regular expression that defines $L$. If such regular expression does not exist, prove it.
**Answer:**

$$((\boldsymbol{bc})^*\boldsymbol{d})^*\boldsymbol{aa}.$$

**Problem 11** Let $L$ be the language generated by the context-free grammar $G = \{V, \Sigma, P, S\}$, where $\Sigma = \{a, b, c, d\}$; $V = \{S, A\}$; and the set of productions $P$ is:

$$\begin{aligned} &S \to aS \mid bS \mid cS \mid dA \\ &A \to AA \mid \lambda \mid a \mid b \mid c \end{aligned}$$

Write a regular expression that defines $L$. If such a regular expression does not exist, explain why.
**Answer:**

$$(\boldsymbol{a} \cup \boldsymbol{b} \cup \boldsymbol{c})^*\boldsymbol{d}(\boldsymbol{a} \cup \boldsymbol{b} \cup \boldsymbol{c})^*$$

**Problem 12** Let $L$ be a language over alphabet $\{a, b\}$ with the following property:

> For every word $w \in L$:
> if $|w|$ is odd, then the middle symbol of $w$ is $a$;
> if $|w|$ is even, then $w$ ends with $bb$.

Write a complete formal definition of a context-free grammar $G$ that generates $L$. If such grammar $G$ does not exist, explain why.
**Answer:** $G = \{V, \Sigma, P, S\}$, where:
$\Sigma = \{a, b\}$ is the set of terminals;
$V = \{S, E, O\}$ is the set of variables;
$S$ is the start symbol;
and the set of productions $P$ comprises:

$$\begin{aligned} &S \to O \mid E \\ &O \to aOa \mid aOb \mid bOa \mid bOb \mid a \\ &E \to aaE \mid abE \mid baE \mid bbE \mid bb \end{aligned}$$

**Problem 13** Let $L$ be the set of strings over alphabet $\{0, 1\}$ in which every 0 is followed immediately by 111.

1. Write a regular expression that defines $L$. If such regular expression does not exist, explain why.

   **Answer:**

   $$(\boldsymbol{1} \cup \boldsymbol{0111})^*$$

2. Write a complete formal definition of a context-free grammar $G$ that generates $L$. If such grammar does not exist, explain why.

   **Answer:** $G = (V, \Sigma, P, S)$, where $\Sigma = \{0, 1\}$,
   $V = \{S\}$, and the production set $P$ is:

   $$S \to SS \mid \lambda \mid 1 \mid 0111$$

3. What is the cardinality of $L$? (If possible, state the exact number. Otherwise, specify if the set is countable or not.) Explain your answer briefly.

   **Answer:**

   $$|L| = \aleph_0$$

   $L$ is infinite and countable. To see that it is infinite, observe that it is the Kleene star of a non-empty set. It is countable, since it is a subset of the set of all finite strings over $\{0, 1\}$, which is countable.

4. What is the cardinality of $\mathcal{P}(L)$—the set of subsets of $L$? (If possible, state the exact number. Otherwise, specify if the set is countable or not.) Explain your answer briefly.

   **Answer:** $\mathcal{P}(L)$ is uncountable—by Cantor's diagonal argument, the set of subsets of an infinite countable set is uncountable.

**Problem 14** **(a)** Let $L_1$ be a language over alphabet $\{a, b, c, d, e\}$, defined as follows:

$$L_1 = \{d^j a^{3n} b^{2m} c^{k+2} d^{2m+1} e^{n+1} d^\ell \mid j, k, \ell, m, n, \geq 0\}$$

Write a complete formal definition of a context-free grammar $G_1$ that generates language $L_1$. If such grammar does not exist, explain why.

**Answer:** $G_1 = \{V, \Sigma, P, S\}$, where:
$\Sigma = \{a, b, c, d, e\}$, $V = \{S, A, B, D, E\}$,
and the production set $P$ is:

$$\begin{aligned}
&S \rightarrow DAD \\
&D \rightarrow dD \mid \lambda \\
&A \rightarrow aaaAe \mid Be \\
&B \rightarrow bbBdd \mid Ed \\
&E \rightarrow cE \mid cc
\end{aligned}$$

**(b)** Let $L_2$ be the set of strings over alphabet $\{a, b, c, d, e\}$ in which the total number of $d$'s and $e$'s is 3.
Write a complete formal definition of a context-free grammar $G_2$ that generates language $L_2$. If such grammar does not exist, explain why.

**Answer:** $G_2 = \{V, \Sigma, P, S\}$, where:
$\Sigma = \{a, b, c, d, e\}$, $V = \{S, A, Z\}$,
and the production set $P$ is:

$$\begin{aligned}
&S \rightarrow AZAZAZA \\
&A \rightarrow AA \mid \lambda \mid a \mid b \mid c \\
&Z \rightarrow d \mid e
\end{aligned}$$

**Problem 15** Write a complete formal definition of a context-free grammar that generates the set of strings over $\{a, b, c\}$ in which the total number of $b$'s and $c$'s is three. If such grammar does not exist, prove it.

**Answer:** $G = \{V, \Sigma, P, S\}$, where $\Sigma = \{a, b, c\}$,
$V = \{S, A, B\}$, and $P$ is:

$$\begin{aligned}
&S \rightarrow ABABABA \\
&A \rightarrow aA \mid \lambda \\
&B \rightarrow b \mid c
\end{aligned}$$

**Problem 16** Let:

$$L = \{a^{2i} b^j d^{3k} a^{\ell+2} \mid i = k, \text{ and } i, j, k, \ell \geq 0\}$$

Write a complete formal definition of a context-free grammar that generates $L$. If such grammar does not exist, prove it.

**Answer:** $G = \{V, \Sigma, P, S\}$, where:
$\Sigma = \{a, b, d\}$, $V = \{S, A, B, D\}$,
and the production set $P$ is:

$$\begin{aligned}
&S \rightarrow AB \\
&A \rightarrow aaAddd \mid D \\
&D \rightarrow bD \mid \lambda \\
&B \rightarrow aB \mid aa
\end{aligned}$$

**Problem 17** Let:

$$L_1 = \{a^i b^j c^k \mid i = 2j \text{ or } j = 2k,\ i,j,k \geq 0\}$$

and

$$L_2 = \{a^i b^j c^k \mid i = 2j \text{ and } j = 2k,\ i,j,k \geq 0\}$$

**(a)** Write a complete formal definition of a context-free grammar that generates $L_1$. If such grammar does not exist, prove it.

**Answer:** Observe that $L_1 = L' \cup L''$ where:

$$\begin{aligned} L' &= \{a^{2j} b^j c^k \mid j,k \geq 0\} \\ L'' &= \{a^i b^{2k} c^k \mid i,k \geq 0\} \end{aligned}$$

$G = \{V, \Sigma, P, S\}$, where
$\Sigma = \{a,b,c\}$, $V = \{S, S', S'', A, B, D, E\}$, and $P$ is:

$$\begin{aligned} &S \to S' \mid S'' \\ &S' \to AB \\ &A \to aaAb \mid \lambda \\ &B \to cB \mid \lambda \\ &S'' \to DE \\ &D \to aD \mid \lambda \\ &E \to bbEc \mid \lambda \end{aligned}$$

**(b)** Write a complete formal definition of a context-free grammar that generates $L_2$. If such grammar does not exist, prove it.

**Answer:** $L_2$ does not have a context-free grammar. Observe that:

$$L_2 = \{a^{4k} b^{2k} c^k \mid k \geq 0\}$$

To prove that $L_2$ is not context-free, assume the opposite—that the Pumping Lemma holds for the language.

Let $\nu$ be the constant as in the Pumping Lemma for $L_2$. Let $w = a^{4m} b^{2m} c^m$, such that $m > \nu$. Let $w = uvxyz$ be a decomposition of $w$, where $vy$ is the pumping part.

Observe that every substring of $w$ that contains all the three letters $(a,b,c)$ must contain the entire run of $b$'s of length $2m$, plus some $a$'s before this run of $b$'s and some $c$'s after it. Hence, every substring of $w$ that contains all the three letters has length at least $2m+2$. By the Lemma, it must be that $|vxy| < \nu < m < 2m+2$. Thus, the pumping part $vy$ is too short to contain all the three letters—it lacks at least one of them, say letter $\xi$, for some $\xi \in \{a,b,c\}$.

After the pumping is admitted, any word of the form $uv^ixy^iz$, for $i > 1$, is claimed to be in $L_2$. However, since the other two letters are pumped, while $\xi$ is not pumped, such a word has a surplus of the other two letters, relative to $\xi$, thereby violating the pattern $a^{4k}b^{2k}c^k$—a contradiction.

**Problem 18** Let $L$ be the set of all strings over $\{a,b,c\}$ that contain $ab$ or $ac$ as a substring. Write a complete formal definition of a context-free grammar that generates $L$. If such grammar does not exist, prove it.

**Answer:** $G = \{V, \Sigma, P, S\}$, where:
$\Sigma = \{a,b,c\}$, $V = \{S, A\}$, and the production set $P$ is:

$$\begin{aligned} &S \to AabA \mid AacA \\ &A \to AA \mid \lambda \mid a \mid b \mid c \end{aligned}$$

**Problem 19** Let $L$ be the language generated by the context-free grammar $G = \{V, \Sigma, P, S\}$, where
$\Sigma = \{a,b\}$, $V = \{S, B, D, E\}$, and $P$ is:

$$\begin{aligned} &S \to aB \mid bD \mid a \\ &B \to bD \mid a \mid b \\ &D \to a \mid b \\ &E \to aS \mid bE \mid \lambda \end{aligned}$$

**(a)** Construct a state-transition graph of a finite automaton that accepts $L$. If such automaton does not exist, prove it.

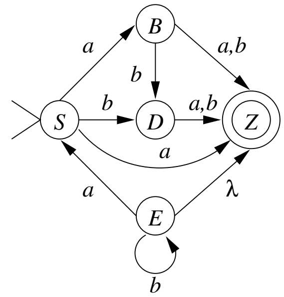


Figure 1:

**Answer:** See Figure 1.

**(b)** Does there exist an algorithm that solves the following problem:

INPUT: An arbitrary context-free grammar $G$.

QUESTION: Is $G$ a regular grammar?

Explain your answer briefly.

**Answer:** Yes—by definition, a grammar is regular exactly when each of its productions has one of the following forms:

$$A \to \lambda$$
$$A \to a$$
$$A \to aB$$

where $A$ and $B$ stand for arbitrary non-terminals, while $a$ stands for an arbitrary terminal.

**Problem 20** Let $L$ be the language defined by the regular expression:

$$(\boldsymbol{abc})^* \cup (\boldsymbol{ab})^*(\boldsymbol{ba})^*$$

**(a)** Draw a state-transition graph of a finite automaton that accepts $L$. If such automaton does not exist, prove it.

**Answer:** See Figure 2.

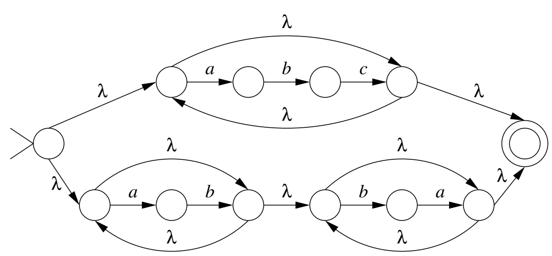

Figure 2:

**(b)** Is the complement $\overline{L}$ of the language $L$ regular? Explain your answer briefly.

**Answer:** Yes—$L$ is regular because it is defined by a regular expression; the complement of any regular language is regular.

**Problem 21** Let $L$ be the language defined by the regular expression

$$(\boldsymbol{a} \cup \boldsymbol{b})^*(\boldsymbol{c} \cup \boldsymbol{a}) \cup (\boldsymbol{ccc}(\boldsymbol{ab} \cup \boldsymbol{cb} \cup \boldsymbol{bb}))^*$$

**(a)** Write 5 distinct strings that belong to $L$. If such strings do not exist, explain why.

**(b)** Write 5 distinct strings over alphabet $\{a, b, c\}$ that do not belong to $L$. If such strings do not exist, explain why.

**Answer:**

| $\in L$ | $\notin L$ |
|---|---|
| $\lambda$ | $cc$ |
| $c$ | $ccc$ |
| $a$ | $ccca$ |
| $ac$ | $cccb$ |
| $cccab$ | $aaab$ |

**(c)** Write a complete formal definition or a state-transition graph of a finite automaton $M$ that accepts $L$. If such automaton does not exist, explain why.

**Answer:** The state transition graph of the finite automaton that accepts $L$ is given on Figure 3.

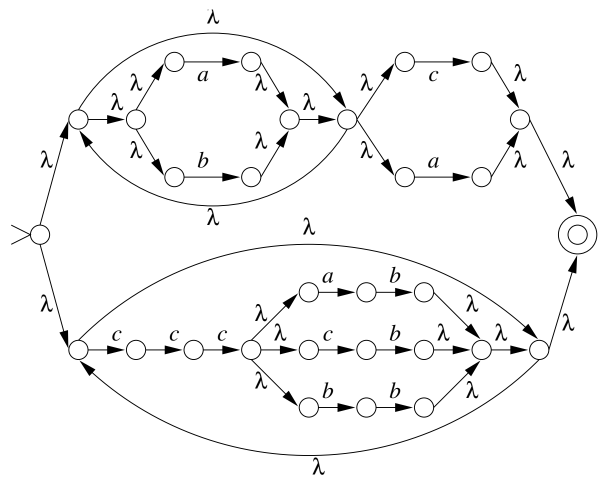

Figure 3:

**Problem 22** Let $L$ be the set of all strings over $\{a, b, c\}$ that end with the substring $abac$. Draw a state-transition graph of a finite automaton that accepts $L$. If such automaton does not exist, prove it.

**Answer:** See Figure 4.

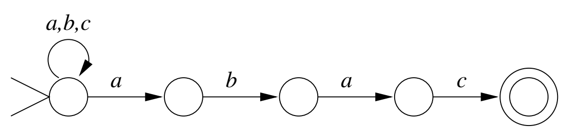

Figure 4:

**Problem 23** Let $L$ be the language over alphabet $\{a, b, c\}$ defined as follows:

$$L = \{c^p b^n a^p \mid n, p \geq 0\}$$

Write a complete formal definition or a state-transition graph of a finite automaton $M$ that accepts $L$. If such automaton does not exist, prove it.

**Answer:** $L$ is not a regular language—there is no finite automaton that accepts it. To prove this, assume the opposite, that $L$ is regular. By the Pumping Lemma, there exists a constant $\ell$ such that every word $w \in L$ such that $|w| \geq \ell$ can be written as

$$w = xyz$$

where:

$$|y| > 0$$
$$|xy| \leq \ell$$

and every word of the form:

$$xy^i z, \ i \geq 0$$

also belongs to $L$. Consider a word $c^k b^m a^k \in L$, where $k$ is chosen so that $k > \ell$. By the Pumping Lemma, there exist $x, y, z$ such that:

$$c^k b^m a^k = xyz$$

where $|xy| \leq \ell < k$, which causes both $x$ and $y$ to consist solely of symbols $c$:

$$y = c^j, \ j > 0$$

and also:

$$xy^i z \in L, \ i > 0$$

meaning:

$$c^{k+(i-1)j}b^m a^k \in L$$

which is impossible by the definition of $L$ whenever $k+(i-1)j \neq k$, which occurs whenever $i > 1$.

**Problem 24** Let $L$ be the set of all strings over $\{a,b\}$ with twice as many $a$'s as $b$'s. Write a complete formal definition or a state-transition graph of a finite automaton $M$ that accepts $L$. If such automaton does not exist, prove it.

**Answer:** Such finite automaton does not exist, since $L$ is not a regular language. To prove this, assume the opposite. Let $k$ be the constant as in the Pumping Lemma. Let $n > k$; then $a^{2n}b^n \in L$. In the "pumping" decomposition: $a^{2n}b^n = uvx$, we have: $|uv| \leq k < n < 2n$, hence the "pumping" substring $v$ consists entirely of $a$'s, say $v = a^j$. Recall that $j > 0$, since the "pumping" substring cannot be empty. By the pumping, every word of the form $uv^ix$, $i \geq 0$, belongs to $L$. However, such a word has exactly $n$ occurrences of $b$ and $2n+(i-1)j$ occurrences of $a$. Since $2n+(i-1)j > 2n$ whenever $i > 1$, this word has too many $a$'s for its number of $b$'s, and this is a contradiction.

**Problem 25** Let:

$$\Sigma = \{a,b,c\}$$

and let $L$ be the set of all strings over $\Sigma$ defined by the following regular expression:

$$\boldsymbol{c(a \cup b)(a \cup b)^* b}$$

**(a)** Construct a state-transition graph of a finite automaton $M$ that accepts $L$. If such automaton does not exist, prove it.

**Answer:** See Figure 5.

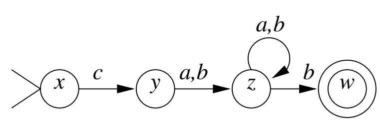


Figure 5:

**(b)** Construct a state-transition graph of a <u>deterministic</u> finite automaton $M'$ that accepts $L$. If such automaton does not exist, prove it.

**Answer:** See Figure 6.

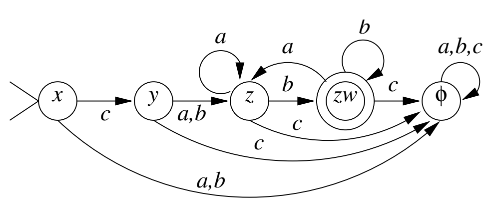


Figure 6:

**Problem 26** Let $L$ be the language accepted by the finite automaton $M = (Q, \Sigma, \delta, q, \{f\})$, where $\Sigma = \{a\}$, $Q = \{f,h,p,q,r,s,t,v,w,x,y,z\}$,
and $\delta$ is given by the following table:

| | $a$ | $\lambda$ |
|---|---|---|
| $f$ | $\emptyset$ | $\{w\}$ |
| $h$ | $\emptyset$ | $\{p\}$ |
| $p$ | $\{z\}$ | $\emptyset$ |
| $q$ | $\{t,r\}$ | $\{s\}$ |
| $r$ | $\emptyset$ | $\{q\}$ |
| $s$ | $\emptyset$ | $\{t,v\}$ |
| $t$ | $\{z,y\}$ | $\{p\}$ |
| $v$ | $\{y\}$ | $\{h\}$ |
| $w$ | $\{x\}$ | $\{r\}$ |
| $x$ | $\{p\}$ | $\{w\}$ |
| $y$ | $\{p\}$ | $\{f\}$ |
| $z$ | $\emptyset$ | $\{w\}$ |

Compute the $\lambda$-closure of state $w$.

**Answer:**

$$\mathcal{C}(w) = \{w, r, q, s, t, v, p, h\}$$

**Problem 27** Let $M$ be the finite automaton represented by the state diagram on Figure 7, and let $L$ be the language accepted by $M$.

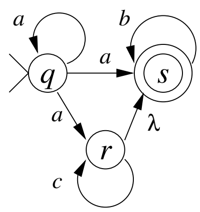


Figure 7:

Construct a state-transition graph of a deterministic finite automaton $M_1$ that accepts $L$, and show your work. If such automaton does not exist, prove it.

**Answer:**

Let $M_1 = (Q', \{a, b, c\}, \delta', q', F')$, where $Q' \in \mathcal{P}(Q)$.

Transition function of $M$:

| $\delta$ | $a$ | $b$ | $c$ | $\lambda$ |
|---|---|---|---|---|
| $q$ | $\{q, s, r\}$ | $\emptyset$ | $\emptyset$ | $\emptyset$ |
| $s$ | $\emptyset$ | $\{s\}$ | $\emptyset$ | $\emptyset$ |
| $r$ | $\emptyset$ | $\emptyset$ | $\{r\}$ | $\{s\}$ |

$\lambda$-closure:

| $q$ | $\mathcal{C}(q)$ |
|---|---|
| $q$ | $\{q\}$ |
| $s$ | $\{s\}$ |
| $r$ | $\{r, s\}$ |

The initial state: $q' = \{q\}$.
The transition function $\delta'$:

| $\delta'$ | $a$ | $b$ | $c$ |
|---|---|---|---|
| $\{q\}$ | $\{q, s, r\}$ | $\emptyset$ | $\emptyset$ |
| $\{q, s, r\}$ | $\{q, s, r\}$ | $\{s\}$ | $\{s, r\}$ |
| $\{s\}$ | $\emptyset$ | $\{s\}$ | $\emptyset$ |
| $\{s, r\}$ | $\emptyset$ | $\{s\}$ | $\{s, r\}$ |
| $\emptyset$ | $\emptyset$ | $\emptyset$ | $\emptyset$ |

The set of states:
$Q' = \{\{q\}, \{s\}, \{s, r\}, \{q, s, r\}, \emptyset\}$.
The set of final states:
$F' = \{\{s\}, \{s, r\}, \{q, s, r\}\}$.
The state diagram of $M_1$ is given on Figure 8.

**Problem 28** Let $M$ be the finite automaton represented by the state diagram on Figure 9, and let $L$ be the language accepted by $M$.

Write a complete formal definition or a state-transition graph of a deterministic finite automaton $M'$ that accepts $L$ and show your work. If such automaton does not exist, prove it.

**Answer:**

Transition function of $M$:

| $\delta$ | $a$ | $b$ | $\lambda$ |
|---|---|---|---|
| $x$ | $\{z\}$ | $\emptyset$ | $\{y, w\}$ |
| $y$ | $\{y\}$ | $\emptyset$ | $\emptyset$ |
| $z$ | $\emptyset$ | $\{y\}$ | $\{w\}$ |
| $w$ | $\{y\}$ | $\emptyset$ | $\emptyset$ |

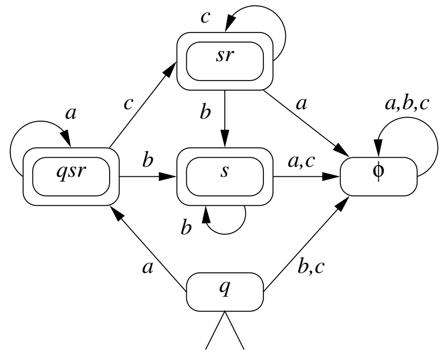


Figure 8:

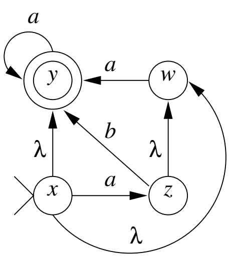


Figure 9:

$\lambda$-closure:

| $q$ | $\mathcal{C}(q)$ |
|---|---|
| $x$ | $\{x, y, w\}$ |
| $y$ | $\{y\}$ |
| $z$ | $\{z, w\}$ |
| $w$ | $\{w\}$ |

The initial state: $q_0' = \mathcal{C}(x) = \{x, y, w\}$.
Input transition function of $M'$:

| $t$ | $a$ | $b$ |
|---|---|---|
| $x$ | $\{z, y, w\}$ | $\emptyset$ |
| $y$ | $\{y\}$ | $\emptyset$ |
| $z$ | $\{y\}$ | $\{y\}$ |
| $w$ | $\{y\}$ | $\emptyset$ |

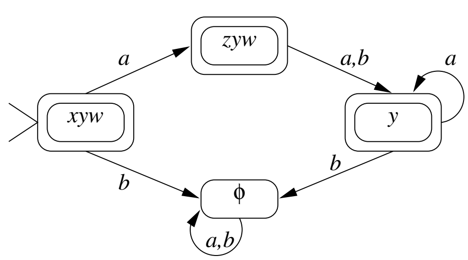


Figure 10:

The transition function $\delta'$:

| $\delta$ | $a$ | $b$ |
|---|---|---|
| $\{x, y, w\}$ | $\{z, y, w\}$ | $\emptyset$ |
| $\{z, y, w\}$ | $\{y\}$ | $\{y\}$ |
| $\{y\}$ | $\{y\}$ | $\emptyset$ |
| $\emptyset$ | $\emptyset$ | $\emptyset$ |

The set of states:
$Q' = \{\{x, y, w\}, \{z, y, w\}, \{y\}, \emptyset\}$.

The set of final states:
$F' = \{\{x, y, w\}, \{z, y, w\}, \{y\}\}$.
The state diagram of $M'$ is given on Figure 10.

**Problem 29** Let $M$ be the finite automaton represented by the state diagram on Figure 11, and let $L$ be the language accepted by $M$.

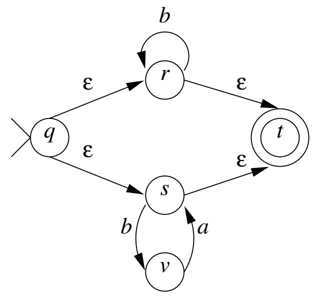


Figure 11:

**(a)** Is the finite automaton $M$ deterministic? Justify briefly your answer.

**Answer:** No—for example, $M$ has $\epsilon$-transitions; there is no transition from state $s$ on symbol $a$, etc.

**(b)** If $M$ is not deterministic, construct a deterministic finite automaton $M'$ that accepts $L$ and show your work. If such an automaton $M'$ does not exist, explain why.

**Answer:** Let $M' = (Q', \{a, b\}, \delta', q', F')$, where $Q' \in \mathcal{P}(Q)$.
Transition function of $M$:

| $\delta$ | $a$ | $b$ | $\epsilon$ |
|---|---|---|---|
| $q$ | $\emptyset$ | $\emptyset$ | $\{r, s\}$ |
| $r$ | $\emptyset$ | $\{r\}$ | $\{t\}$ |
| $s$ | $\emptyset$ | $\{v\}$ | $\{t\}$ |
| $t$ | $\emptyset$ | $\emptyset$ | $\emptyset$ |
| $v$ | $\{s\}$ | $\emptyset$ | $\emptyset$ |

$\epsilon$-closure:

| $x$ | $\mathcal{C}(x)$ |
|---|---|
| $q$ | $\{q, r, s, t\}$ |
| $r$ | $\{r, t\}$ |
| $s$ | $\{s, t\}$ |
| $t$ | $\{t\}$ |
| $v$ | $\{v\}$ |

The initial state: $q' = \mathcal{C}(q) = \{q, r, s, t\}$.
The transition function $\delta'$:

| $\delta'$ | $a$ | $b$ |
|---|---|---|
| $\{q, r, s, t\}$ | $\emptyset$ | $\{r, t, v\}$ |
| $\{r, t, v\}$ | $\{s, t\}$ | $\{r, t\}$ |
| $\{s, t\}$ | $\emptyset$ | $\{v\}$ |
| $\{r, t\}$ | $\emptyset$ | $\{r, t\}$ |
| $\{v\}$ | $\{s, t\}$ | $\emptyset$ |
| $\emptyset$ | $\emptyset$ | $\emptyset$ |

The set of states:
$Q' = \{\{q, r, s, t\}, \{r, t, v\}, \{s, t\}, \{r, t\}, \{v\}, \emptyset\}$.
The set of final states:
$F' = \{\{q, r, s, t\}, \{r, t, v\}, \{s, t\}, \{r, t\}\}$.
The state diagram of $M'$ is given on Figure 12.

**Problem 30** Let $M$ be the finite automaton represented by the state diagram on Figure 13, and let $L$ be the language accepted by $M$.

Construct a regular expression that defines $L$ and show your work. If such regular expression does not exist, prove it.

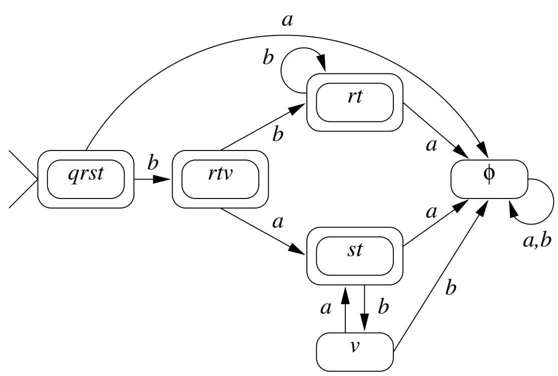


Figure 12:

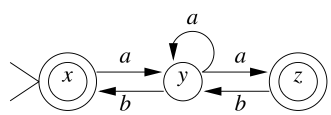


Figure 13:

**Answer:**

$$\boldsymbol{a(a \cup ab \cup ba)^*(a \cup b) \cup \lambda}$$

The sequence of expression graphs equivalent to $M$ is represented on Figure 14.

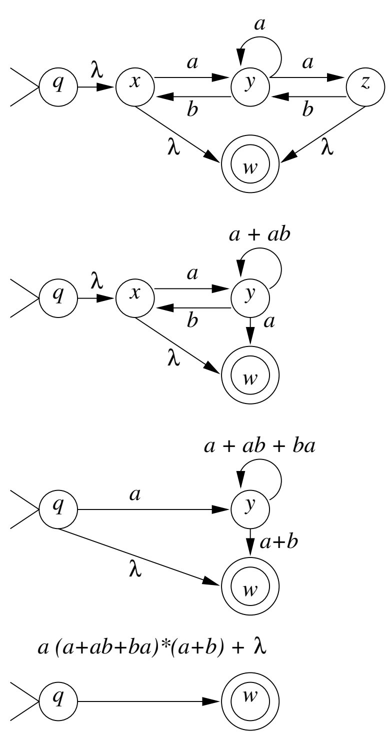


Figure 14:

**Problem 31** Let $M$ be the finite automaton represented by the state diagram on Figure 15, and let $L$ be the language accepted by $M$.

Construct a regular expression that defines $L$ and show your work. If such a regular expression does not exist, explain why.

**Answer:**

$$\boldsymbol{(a \cup b)a^*b(a(a \cup b)a^*b \cup ba^*b)^* \cup \epsilon}$$

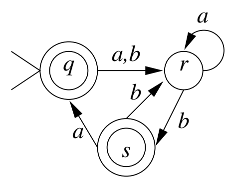


Figure 15:

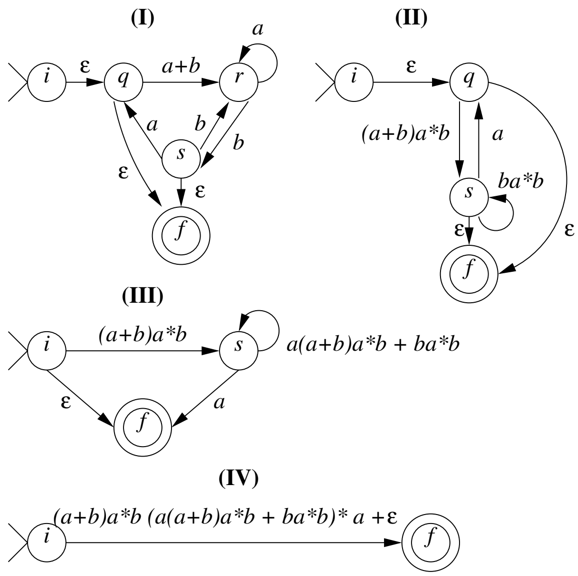


Figure 16:

The sequence of generalized finite automata equivalent to $M$ is represented on Figure 16.