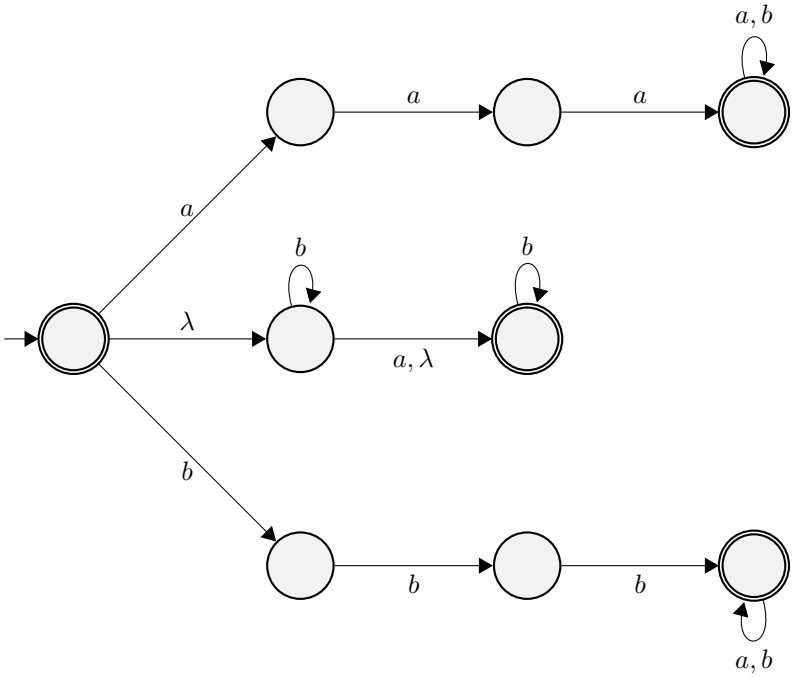
QUEENS COLLEGE Department of Computer Science
CSCI 320 Midterm 1 Exam Fall 2024 10.30.24
Instructor: Alex Ryba

09.15am – 10.30am, Wednesday, October 30, 2024

Problem 1 **(a)** Let L_1 be the set of all strings over alphabet $\{a, b\}$ that contain exactly three copies of a.
Write a regular expression that defines L_1. **Answer:** $b^*ab^*ab^*ab^*$
(b) Let L_2 be the set of all strings over alphabet $\{a, b\}$ that contain at least three copies of a.
Write a regular expression that defines L_2. **Answer:** $b^*ab^*ab^*a(a \cup b)^*$
(c) Let L_3 be the set of all strings over alphabet $\{a, b\}$ that contain at most three copies of a.
Write a regular expression that defines L_3. **Answer:** $b^*(a \cup \lambda)b^*(a \cup \lambda)b^*(a \cup \lambda)b^*$

Problem 2 Let L_1 be the set of all strings over alphabet $\{a, b\}$ that have length at least 3 and whose first three letters are equal. Let L_2 be the set of strings over alphabet $\{a, b\}$ that contain at most one a.
(a) Draw a state-transition graph of a finite automaton that accepts $L_1 \cup L_2$.
Answer:



(b) Give a regular expressions that defines $L_1 \cap L_2$. **Answer:** $bbbb^*(a \cup \lambda)b^*$
(c) Write a complete definition of a context-free grammar that generates L_1L_2.
Answer: The states of a grammar are S, L_1, L_2. Its rules are:

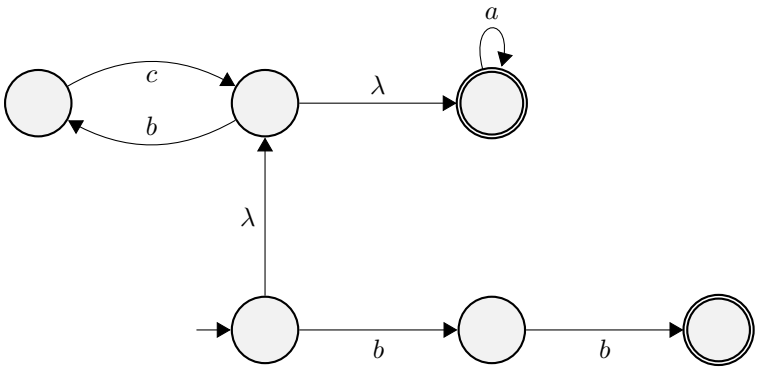
- $S \to L_1L_2$
- $L_1 \to aaa|bbb|L_1a|L_1b$
- $L_2 \to a|\lambda|bL_2|L_2b$

Problem 3 Let L be the language defined by the regular expression:

$$(\boldsymbol{bc})^*\boldsymbol{a}^* \cup \boldsymbol{bb}$$

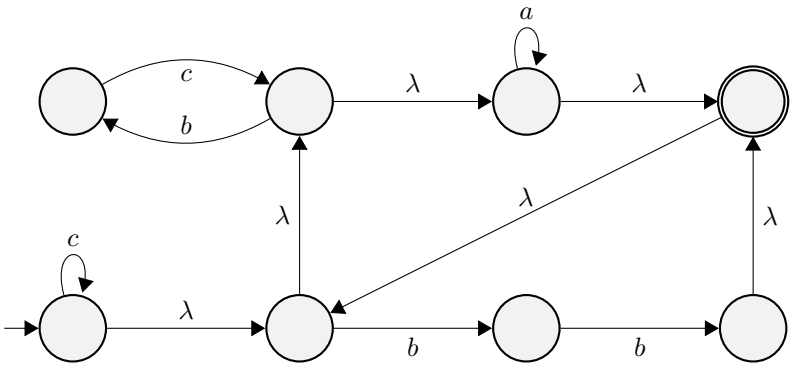
(a) Construct a state-transition graph of a finite automaton that accepts L.

Answer:

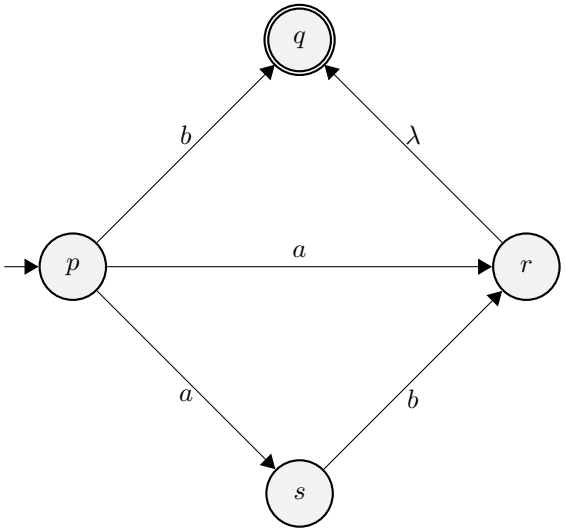


(b) Construct a state-transition graph of a finite automaton that accepts c^*L^*.

Answer:



Problem 4 Let L be the language accepted by the NFA M with the following state transition graph.

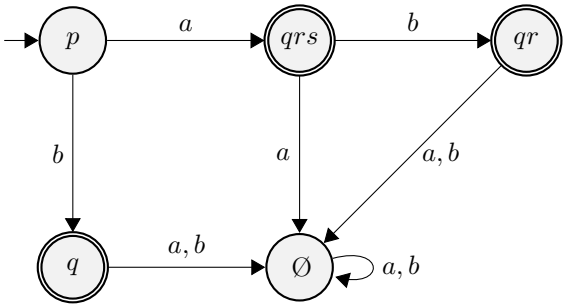


(a) What are the λ-closures of the state s and the set $\{p, r\}$? **Answer:** $\{s\}$ and $\{p, q, r\}$

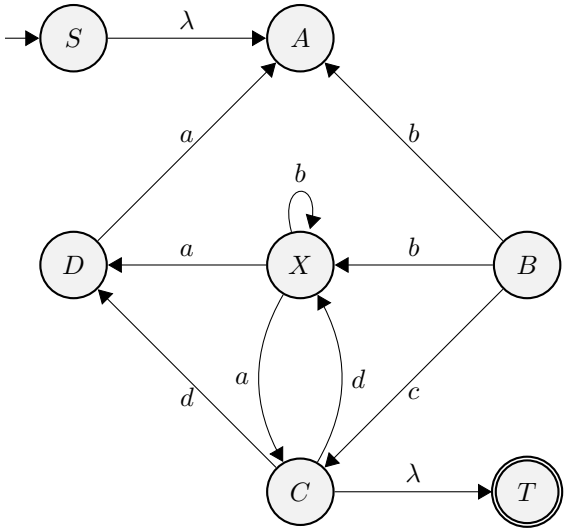
(b) Suppose that M is either in state p or state s. Write the set of all possible states that M could reach after it has processed only the character a from an input string. **Answer:** $\{q, r, s\}$

(c) Draw a state-transition graph of a deterministic finite automaton that accepts L. The states of your automaton must be labelled in a meaningful way by sets of states of M.

Answer:



Problem 5 Consider the following finite automaton.



(a) Which edge labels would need to be changed when we replace M by an equivalent regular expression machine that is obtained by eliminating the node X. **Answer:** BD, BC, CD, CC.

(b) What would be the new label on the edge BC? **Answer:** $c \cup bb^*a$

(c) Draw a diagram of the regular expression machine obtained from this automaton when the node X is eliminated using one step of the algorithm for conversion of a finite automaton to a regular expression. (Only show how to remove the node X. Do not complete the algorithm to obtain a regular expression that corresponds to the automaton.) **Answer:**