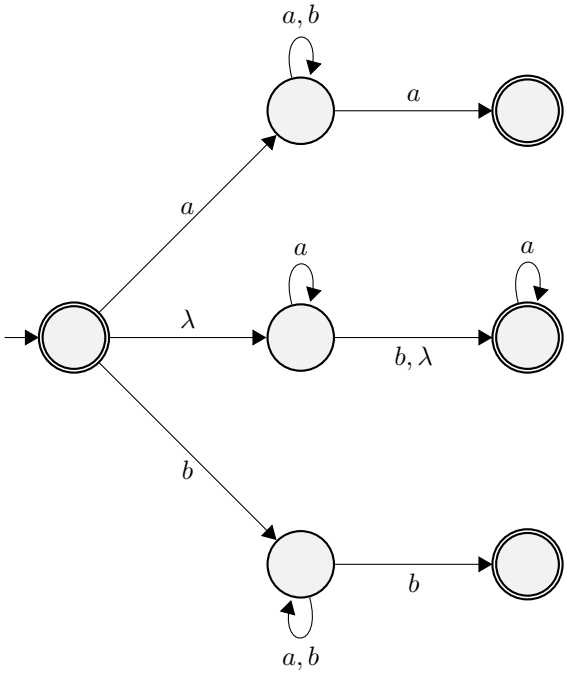
QUEENS COLLEGE Department of Computer Science
CSCI 320 Midterm 1 Exam Fall 2024 10.30.24
Instructor: Alex Ryba

09.15am – 10.30am, Wednesday, October 30, 2024

Problem 1 **(a)** Let L_1 be the set of all strings over alphabet $\{0,1\}$ that contain exactly two zeros.
Write a regular expression that defines L_1. **Answer:** $1^*01^*01^*$
(b) Let L_2 be the set of all strings over alphabet $\{0,1\}$ that contain at least two zeros.
Write a regular expression that defines L_2. **Answer:** $1^*01^*0(0 \cup 1)^*$
(c) Let L_3 be the set of all strings over alphabet $\{0,1\}$ that contain at most two zeros.
Write a regular expression that defines L_3. **Answer:** $1^*(0 \cup \lambda)1^*(0 \cup \lambda)1^*$

Problem 2 Let L_1 be the set of all strings over alphabet $\{a,b\}$ that have length at least 2 and whose first and last letters are equal. Let L_2 be the set of strings over alphabet $\{a,b\}$ that contain at most one b.
(a) Draw a state-transition graph of a finite automaton that accepts $L_1 \cup L_2$.
Answer:



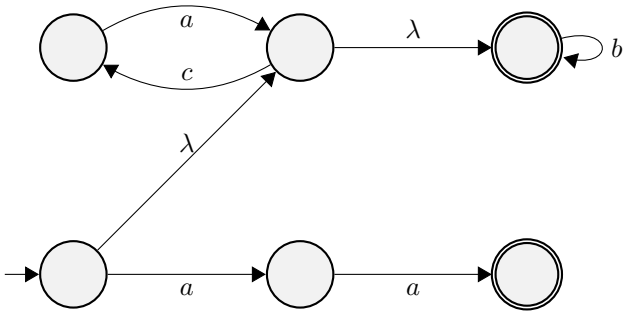
(b) Give a regular expressions that defines $L_1 \cap L_2$. **Answer:** $aa^*(b \cup \lambda)a^*a$
(c) Write a complete definition of a context-free grammar that generates L_1L_2.
Answer: The grammar has symbols S, L_1, L_2, X and rules:

- $S \to L_1L_2$
- $L_1 \to aXa|bXb$
- $X \to \lambda|aX|bX$
- $L_2 \to \lambda|b|aL_2|L_2a$

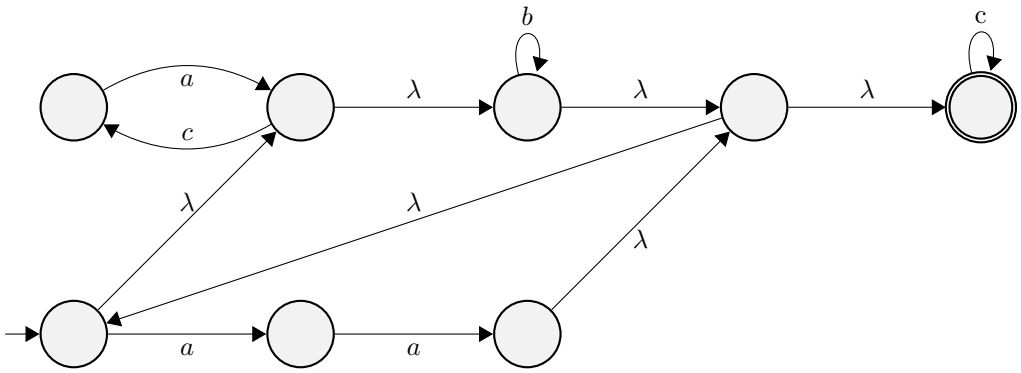
Problem 3 Let L be the language defined by the regular expression:

$$(ca)^*b^* \cup aa$$

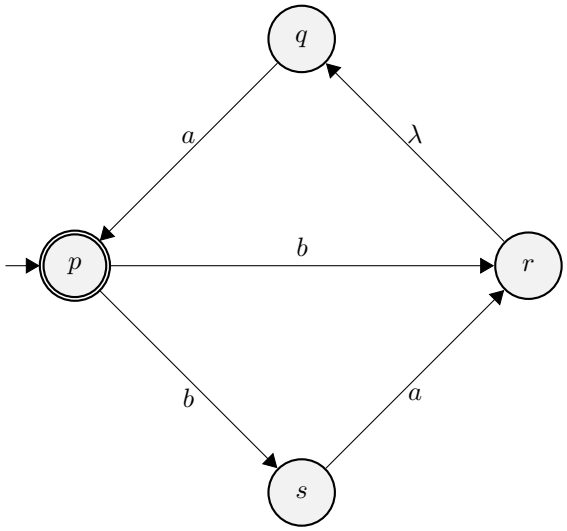
(a) Construct a state-transition graph of a finite automaton that accepts L.
Answer:



(b) Construct a state-transition graph of a finite automaton that accepts L^*c^*.
Answer:



Problem 4 Let L be the language accepted by the NFA M with the following state transition graph.

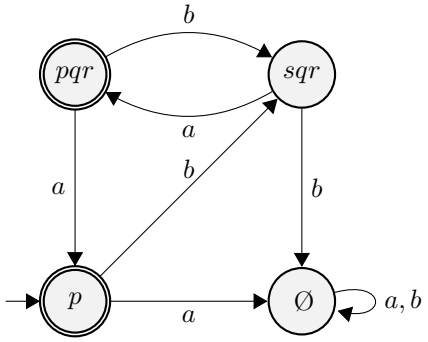


(a) What are the λ-closures of the state s and the set $\{p, r\}$? **Answer:** $\{s\}$ and $\{p, q, r\}$

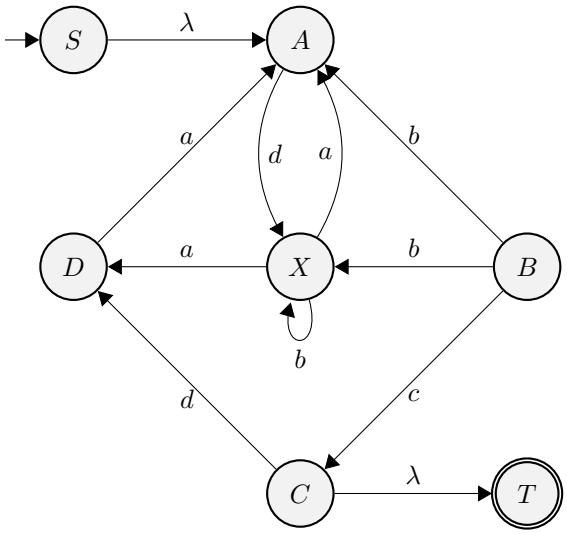
(b) Suppose that M is either in state p or state s. Write the set of all possible states that M could reach after it has processed only the character a from an input string. **Answer:** $\{q, r\}$

(c) Draw a state-transition graph of a deterministic finite automaton that accepts L. The states of your automaton must be labelled in a meaningful way by sets of states of M.

Answer:



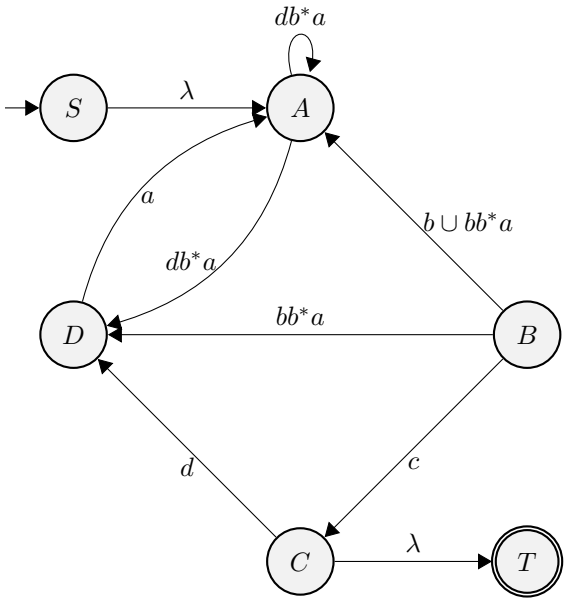
Problem 5 Consider the following finite automaton.



(a) Which edge labels would need to be changed when we replace M by an equivalent regular expression machine that is obtained by eliminating the node X. **Answer:** AA, AD, BA, BD.

(b) What would be the new label on the edge BA? **Answer:** $b \cup bb^*a$

(c) Draw a diagram of the regular expression machine obtained from this automaton when the node X is eliminated using one step of the algorithm for conversion of a finite automaton to a regular expression. (Only show how to remove the node X. Do not complete the algorithm to obtain a regular expression that corresponds to the automaton.) **Answer:**


db^*a
S
λ
A
a
db^*a
$b \cup bb^*a$
D
bb^*a
B
d
c
C
λ
T