Problem 1  Give useful $\Theta$ estimates for the following functions $t(n)$.

(a) $t(n) = 5 \log_2(n^2) + (\log_2(n))^2 + \log_4(n) + (\log_2(100))^3$.
   
   **Answer:**
   
   Simple $\Theta$ estimates for the terms are $\log(n)$, $(\log(n))^2$, $\log(n)$ and 1. The second of these grows fastest so our estimate is $t(n) = \Theta((\log(n))^2)$.

(b) $t(n)$ satisfies $t(n) = 2t(n/2) + n$.
   
   **Answer:**
   
   The recursive and nonrecursive costs are both $\Theta(n)$ so the Master Theorem gives $t(n) = \Theta(n\log(n))$.

(c) $t(n)$ satisfies $t(n) = 4t(n/3) + n$.
   
   **Answer:**
   
   The recursive cost is $\Theta(n^{\log_4(3)})$ which grows faster than the nonrecursive cost of $\Theta(n)$. Hence the Master Theorem gives $t(n) = \Theta(n^{\log_4(3)})$.

(d) $t(n)$ is the running time of the following function:

```java
public static void shuffle(int []x, int a, int b, int n) {
    for (int i = 0; i < n; i+=2) {
        int temp = x[a + i];
        x[a + i] = x[b + i];
        x[b + i] = temp;
    }
}
```

**Answer:**

The function performs $n/2$ iterations of a loop and each iteration has constant running time. Hence $t(n) = \Theta(n)$.

(e) $t(n)$ is the running time of the following function that calls shuffle from (d):

```java
public static void multiShuffle(int []x, int a, int n) {
    if (n == 0) return;
    multiShuffle(x, a, n/2);
    multiShuffle(x, a + n/4, n/2);
    multiShuffle(x, a + n/2, n/2);
    shuffle(x, a, a + n/2, n/2);
}
```

**Answer:**

Here, the recursive cost is $\Theta(n^{\log_3(3)})$ which grows faster than the nonrecursive cost of $\Theta(n)$ (given by (d)). Hence the Master Theorem gives $t(n) = \Theta(n^{\log_3(3)})$.

Problem 2  Give useful $O$-estimates of the run times of the following methods:

(a) The method `addHead` for a singly linked list that has size $n$.
   
   **Answer:** $O(1)$

(b) An efficient method to calculate the power $x^n$ (consider the run time as a function of $n$, the time should be considered as being proportional to the total number of additions, subtractions, multiplications, and divisions performed).
   
   **Answer:** $O(\log(n))$

(c) An efficient method to sort an array of $n$ numbers into order.
   
   **Answer:** $O(n\log(n))$

For (d) and (e), consider the following recursive function, in which $A$ represents an integer constant:
int f(int n) {
    if (n <= 0) return 1;
    int ans = f(n/2) * 2;
    for (int i = 1; i<= n; i++)
        for (int j = 1; j <= n; j++)
            ans += i / j;
    for (int k = 1; k < A; k++)
        ans -= f(n/2 - k);
    return ans;
}

(d) In the case where \( A = 3 \) estimate the run time of \( f(n) \).
\textbf{Answer:} \( O(n^2) \)

(e) In the case where \( A = 4 \) estimate the run time of \( f(n) \).
\textbf{Answer:} \( O(n^2 \log(n)) \)

\textbf{Problem 3} Give fully simplified \( \Theta \) estimates for the following functions \( t(n) \).
(a) \( t(n) = 10\sqrt{n} + \log_{17}(17^n) + \log_2(\log_5(2^n)) + 0.01n^{0.54} + \log_2(2^n) \).
\textbf{Answer:} \( \Theta(n) \)

(b) \( t(n) \) is the running time for the \textit{push} method applied to a Stack (with an array based implementation) that contains \( n \) items.
\textbf{Answer:} \( \Theta(1) \)

(c) \( t(n) \) satisfies \( t(n) = 12(t(n/2) + (n - 1)n(2n + 1)) \).
\textbf{Answer:} \( \Theta(n \log_2 12) \)

(d) \( t(n) \) is the running time for \textit{levelOrder} traversal of a tree that contains \( n \) nodes.
\textbf{Answer:} \( \Theta(n) \)

(e) \( t(n) \) is the running time for the \textit{removeMin} method of a priority queue that contains \( n \) items.
\textbf{Answer:} \( \Theta(\log n) \)