Problem 1 (10 points)
In (a) and (b) give useful Θ estimates for the function t(n).
(a) t(n) is the running time for the preOrder method applied to a binary tree that contains n nodes.
Answer: Θ(n)
(b) t(n) satisfies t(n) = 10t(n/2) + (n-1)n(n+1)/6.

Answer: $\Theta(n^{\log_2 10})$

For the following binary tree:

(c) What is the height of the node that stores X?

Answer: 3

(d) What is the depth of the node that stores X?

Answer: 1

(e) What is the postorder traversal?

Answer: A N S M V X B C Z

Problem 2 (10 points) The following is part of a circular array based model for a Deque. The method addLast has logic errors and does not operate correctly. Other required methods have been entirely omitted.

```
public class Deque<T> {
 private T data[];
 private int front, rear, size;
 // the next items to be added at front and rear will be placed at indices front and rear
 public Deque() {
                   // This is correct code.
   data = (T[]) new Object[100];
   front = size = 0; rear = 1;
 }
 public T addLast(T x) throws Exception { // This has errors.
   ++size;
    if (size > 100) throw Exception("Deque Full");
   data[++rear] = x;
    if (rear > 100) rear = 0;
 }
 // other methods omitted
```

}

(a) Correct the code for addLast. Your solution should not use more more than one additional line of code and should be based on that given. It must correct any errors.

Answer:

```
public void addLast(T x) throws Exception {
    if (size >= 100) throw new Exception("Deque Full");
    data[rear++] = x;
    if (rear >= 100) rear = 0;
    ++size;
}
```

(b) Give a title line for each required method that has been omitted. (Credit will be given for the 3 most important methods only.)

Answer:

```
public void addFirst(T x) throws Exception {
  public T removeFirst() throws Exception {
    public T removeLast() throws Exception {
```

Problem 3 (10 points) The class BNode implements a generic binary tree node. The class provides the following instance variables and methods.

```
public class BNode<T> {
   BNode<T> parent, left, right;
   T data;
   public BNode(T data, BNode<T> parent, BNode<T> left, BNode<T> right) // code omitted
   // getters and setters for all instance variables, code omitted
   public void leftShift() // to be written
```

```
}
```

Write complete code for the method *leftShift* that acts on the subtree of descendants of the BNode. For any node lower in this subtree that has only a right child, that right child should be moved to become a left child.

For example, if node.leftShift() is called at the node storing X in the tree of the left hand diagram, the result is the tree in the right hand diagram.

Z	Z
/ \	/ \
X C	X C
\land \land	/ \
V B	V B
/ \	/ \
N M	N M
/ \	/ /
A S	A S

Code longer than 15 lines may be subject to a penalty. Code with a running time worse than O(n) (where n is the number of nodes in the subtree being adjusted) will be subject to a penalty.

Answer:

```
public void leftShift() {
    if (left == null && right == null) return; // base case
    if (left == null) {
        left = right;
        right = null;
    }
    left.leftShift();
    if (right != null) right.leftShift();
}
```