QUEENS COLLEGE Department of Computer Science
CSCI 313 Practice problems on O, Omega, Theta and the Master theorem

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These problems were given on exams for this course. Some older problems did not make use of generics in Java, but generic implementations are now required in this course.

**Problem 1** Give useful $\Theta$ estimates for the following functions $t(n)$.

(a) $t(n) = 5log_2(n^2) + (log_2(n))^2 + log_4(n) + (log_2(100))^3$.

(b) $t(n)$ satisfies $t(n) = 2t(n/2) + n$.

(c) $t(n)$ satisfies $t(n) = 4t(n/3) + n$.

(d) $t(n)$ is the running time of the following function:

```
public static void shuffle(int []x, int a, int b, int n) {
   for (int i = 0; i < n; i+=2) {
      int temp = x[a + i];
      x[a + i] = x[b + i];
      x[b + i] = temp;
   }
}
```

(e) $t(n)$ is the running time of the following function that calls shuffle from (d):

```
public static void multiShuffle(int []x, int a, int n) {
   if (n == 0) return;
   multiShuffle(x, a, n/2);
   multiShuffle(x, a + n/4, n/2);
   multiShuffle(x, a + n/2, n/2);
   shuffle(x, a, a + n/2, n/2);
}
```

**Problem 2** Give useful $O$-estimates of the run times of the following methods:

(a) The method *addHead* for a singly linked list that has size $n$.

(b) An efficient method to calculate the power $x^n$ (consider the run time as a function of $n$, the time should be considered as being proportional to the total number of additions, subtractions, multiplications, and divisions performed).

(c) An efficient method to sort an array of $n$ numbers into order.

For (d) and (e), consider the following recursive function, in which $A$ represents an integer constant:

```
int f(int n) {
   if (n <= 0) return 1;
   int ans = f(n/2) * 2;
   for (int i = 1; i<= n; i++)
      for (int j = 1; j <= n; j++)
         ans += i / j;
   for (int k = 1; k < A; k++)
         ans -= f(n/2 - k);
   return ans;
}
```

(d) In the case where $A = 3$ estimate the run time of $f(n)$.

(e) In the case where $A = 4$ estimate the run time of $f(n)$.

**Problem 3** Give useful O estimates for the run times of the following methods.

(a) *removeMin* for a *PriorityQueue* storing $n$ items in a heap implementation.

(b) *preOrder* for a general *Tree* storing $n$ items.

(c) *get* for a chained *HashTable* storing $n$ items with load factor $\lambda$.

(d) A recursive method $f$ that processes $n$ input items by: sorting the items (efficiently), makes two recursive calls to process $n/2$ items, computes the products of all pairs of input items and finally makes two further recursive calls to process $n/2$ items.