CS320: Problems and Solutions for Day 8, Winter 2023

Problem 1 Let M be the finite automaton represented by the state diagram on Figure 1, and let L be the language accepted by M.

Write a complete formal definition or a state-transition graph of a deterministic finite automaton M' that accepts L and show your work. If such automaton does not exist, prove it.

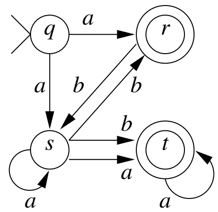

Figure 1:

Answer: There are no λ-transitions; therefore, the λ-closure of every state is the singleton containing that state. Hence, $q' = \{q\}$. Furthermore, the transition function δ of M and the input transition function t of M' are identical:

$t = \delta$	a	b
q	$\{r, s\}$	$\emptyset$
r	$\emptyset$	$\{s\}$
s	$\{s, t\}$	$\{r, t\}$
t	$\{t\}$	$\emptyset$

The transition function δ':

δ'	a	b
$\{q\}$	$\{r, s\}$	$\emptyset$
$\{r, s\}$	$\{s, t\}$	$\{r, s, t\}$
$\{s, t\}$	$\{s, t\}$	$\{r, t\}$
$\{r, s, t\}$	$\{s, t\}$	$\{r, s, t\}$
$\{r, t\}$	$\{t\}$	$\{s\}$
$\{t\}$	$\{t\}$	$\emptyset$
$\{s\}$	$\{s, t\}$	$\{r, t\}$
$\emptyset$	$\emptyset$	$\emptyset$

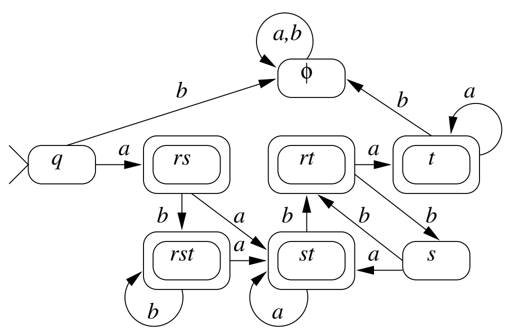

Figure 2:

The set of states:
$Q' = \{\{q\}, \{r, s\}, \{s, t\}, \{r, s, t\}, \{r, t\}, \{t\}, \{s\}, \emptyset\}$.
The set of final states:
$F' = \{\{r, s\}, \{s, t\}, \{r, s, t\}, \{r, t\}, \{t\}\}$.
The state diagram of M' is given on Figure 2.

Problem 2 Let M be the finite automaton represented by the state diagram on Figure 3, and let L be the language accepted by M.

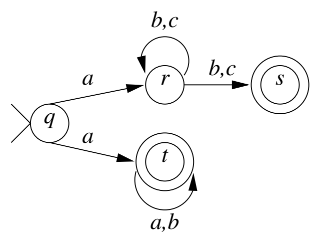

Figure 3:

(a) Is the finite automaton M deterministic? Justify briefly your answer.

Answer: No—for example, there are two states, r and s, reachable from state r on input symbol b (or c), etc.

(b) If M is not deterministic, construct a deterministic finite automaton M' that accepts L and show your work. If such an automaton M' does not exist, explain why.

Answer: Let $M' = (Q', \{a, b, c\}, \delta', q', F')$, where $Q' \in \mathcal{P}(Q)$.

There are no ϵ-transitions; therefore, the ϵ-closure of every state is the singleton containing that state. Hence, $q' = \{q\}$. The transition function δ:

δ	a	b	c
q	$\{r, t\}$	$\emptyset$	$\emptyset$
r	$\emptyset$	$\{r, s\}$	$\{r, s\}$
s	$\emptyset$	$\emptyset$	$\emptyset$
t	$\{t\}$	$\{t\}$	$\emptyset$

The transition function δ':

δ'	a	b	c
$\{q\}$	$\{r, t\}$	$\emptyset$	$\emptyset$
$\{r, t\}$	$\{t\}$	$\{r, s, t\}$	$\{r, s\}$
$\{t\}$	$\{t\}$	$\{t\}$	$\emptyset$
$\{r, s, t\}$	$\{t\}$	$\{r, s, t\}$	$\{r, s\}$
$\{r, s\}$	$\emptyset$	$\{r, s\}$	$\{r, s\}$
$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$

The set of states:
$Q' = \{\{q\}, \{r, t\}, \{t\}, \{r, s, t\}, \{r, s\}, \emptyset\}$.
The set of final states:
$F' = \{\{t\}, \{r, t\}, \{r, s, t\}, \{r, s\}\}$.
The state diagram of M' is given on Figure 4.

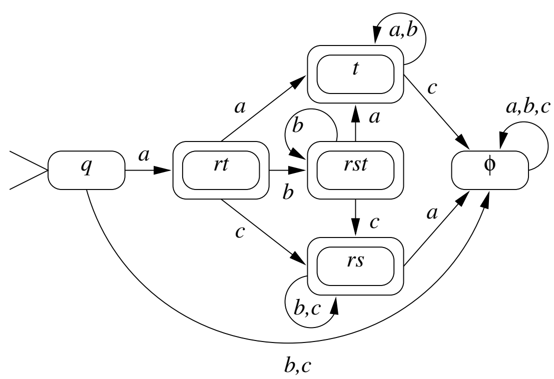

Figure 4:

Problem 3 Let L be the language defined by the regular expression

$$\boldsymbol{b(a \cup b^*((c^* \cup (cb)^*)ac)^*)b}$$

(a) Construct a finite automaton M that accepts L. If such an automaton M does not exist, explain why.

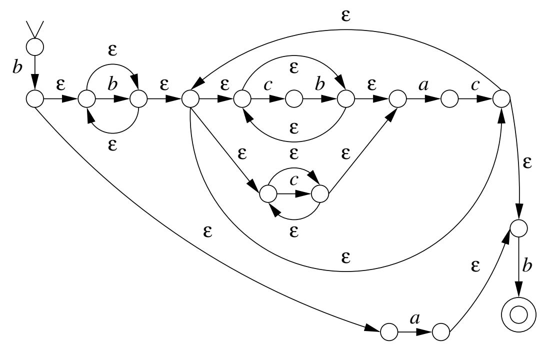

Figure 5:

Answer: The state diagram of M is given on Figure 5.

(b) If you constructed an automaton M in your answer to part (a), is M deterministic? Justify briefly your answer.

Answer: No—for example, M has ϵ transitions, etc.

Problem 4 Let M be the finite automaton represented by the state diagram on Figure 6, and let L be the language accepted by M.

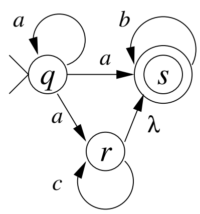

Figure 6:

Construct a state-transition graph of a deterministic finite automaton M_1 that accepts L, and show your work. If such automaton does not exist, prove it.

Answer:

Let $M_1 = (Q', \{a, b, c\}, \delta', q', F')$, where $Q' \in \mathcal{P}(Q)$.

Transition function of M:

δ	a	b	c	λ
q	$\{q, s, r\}$	$\emptyset$	$\emptyset$	$\emptyset$
s	$\emptyset$	$\{s\}$	$\emptyset$	$\emptyset$
r	$\emptyset$	$\emptyset$	$\{r\}$	$\{s\}$

λ-closure:

q	$\mathcal{C}(q)$
q	$\{q\}$
s	$\{s\}$
r	$\{r, s\}$

The initial state: $q' = \{q\}$.
The transition function δ':

δ'	a	b	c
$\{q\}$	$\{q, s, r\}$	$\emptyset$	$\emptyset$
$\{q, s, r\}$	$\{q, s, r\}$	$\{s\}$	$\{s, r\}$
$\{s\}$	$\emptyset$	$\{s\}$	$\emptyset$
$\{s, r\}$	$\emptyset$	$\{s\}$	$\{s, r\}$
$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$

The set of states:
$Q' = \{\{q\}, \{s\}, \{s, r\}, \{q, s, r\}, \emptyset\}$.
The set of final states:
$F' = \{\{s\}, \{s, r\}, \{q, s, r\}\}$.
The state diagram of M_1 is given on Figure 7.

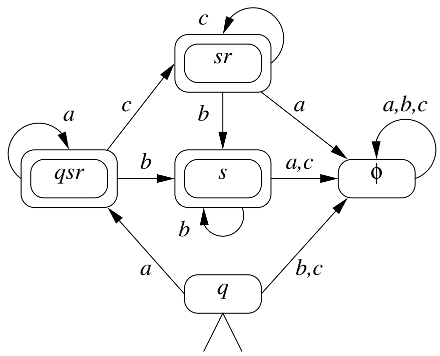

Figure 7:

Problem 5 Let L be the language accepted by the finite automaton $M = (Q, \Sigma, \delta, q, \{f\})$, where $\Sigma = \{a\}$, $Q = \{p, q, r, s, t, v, w, x, y, z, f\}$,
and δ is given by the following table:

	a	λ
p	$\{z\}$	$\emptyset$
q	$\{t, r\}$	$\{s\}$
r	$\emptyset$	$\{q, t\}$
s	$\emptyset$	$\{w\}$
t	$\{z, y\}$	$\{p, w\}$
v	$\{x\}$	$\{r\}$
w	$\{y\}$	$\emptyset$
x	$\{p\}$	$\{v\}$
y	$\{p\}$	$\{f\}$
z	$\emptyset$	$\{v\}$
f	$\emptyset$	$\emptyset$

Compute the λ-closure of state v.

Answer:

$$\mathcal{C}(v) = \{v, r, q, t, s, p, w\}$$

Problem 6 Let M be the finite automaton represented by the state diagram on Figure 8, and let L be the language accepted by M.

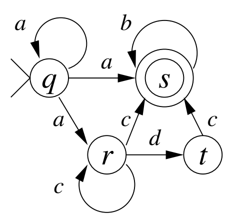

Figure 8:

Write a complete formal definition of a context-free grammar G that generates L. If such grammar does not exist, prove it.

Answer: Apply the algorithm to convert M into a regular context-free grammar: $G = \{V, \Sigma, P, q\}$, where $\Sigma = \{a, b, c\}$, $V = \{q, s, r, t\}$, and the production set P is:

$$\begin{aligned} &q \to aq \mid as \mid ar \\ &s \to bs \mid \lambda \\ &r \to cr \mid cs \mid dt \\ &t \to cs \end{aligned}$$