CS320: Problems and Solutions for Day 6, Winter 2023

Problem 1 Let L_1 be the set of strings over alphabet $\{a, b, c\}$ in which the total number of b's and c's is one. Let L_2 be the set of strings over alphabet $\{a, b, c\}$ that contain an even number of b's.

(a) Draw a state-transition graph of a finite automaton M_1 that accepts $L_1 \cup L_2$. If such automaton does not exist, prove it.

Answer: See Figure 1.

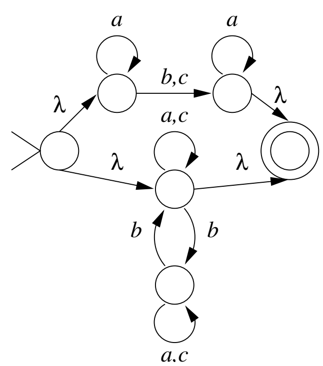

Figure 1:

(b) Draw a state-transition graph of a finite automaton M_2 that accepts $(L_1L_2)^*$. If such automaton does not exist, prove it.

Answer: See Figure 2.

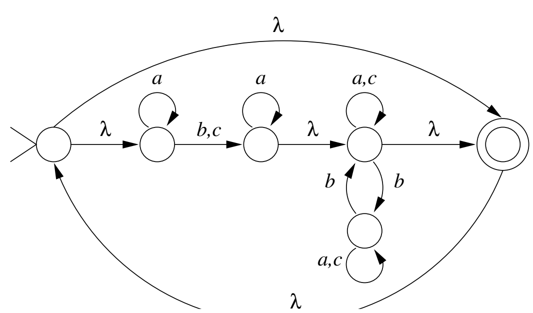

Figure 2:

(c) Is $L_1 \cap L_2$ a context-free language? Explain your answer.

Answer: Yes—$L_1 \cap L_2$ is regular because the intersection of any two regular languages is regular. Every regular language is context-free.

Problem 2 Let L_1 be the set of strings over alphabet $\{a, b, c\}$ that contain at least one b. Let L_2 be the set of strings over alphabet $\{a, b, c\}$ whose length gives remainder 2 when divided by 3.

(a) Draw a state-transition graph of a finite automaton that accepts L_1. If such automaton does not exist, prove it.

Answer: See Figure 3.

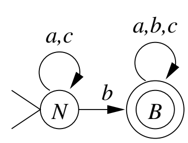

Figure 3:

(b) Draw a state-transition graph of a finite automaton that accepts L_2. If such automaton does not exist, prove it.

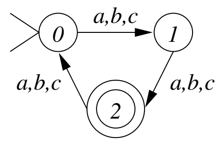

Figure 4:

Answer: See Figure 4.

(c) Draw a state-transition graph of a finite automaton that accepts $L_1 \cup L_2$. If such automaton does not exist, prove it.

Answer: See Figure 5.

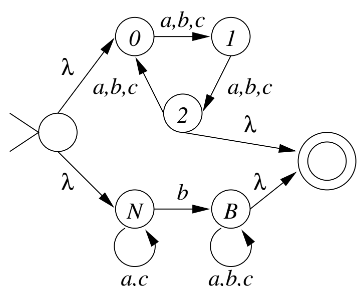

Figure 5:

(d) Draw a state-transition graph of a finite automaton that accepts $L_1 \cap L_2$. If such automaton does not exist, prove it.

Answer: See Figure 6.

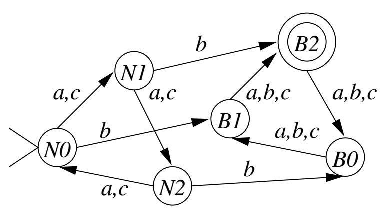

Figure 6:

Problem 3 Let L_1 be the set of all strings over alphabet $\{a, b\}$ whose first and last letters are equal. Let L_2 be the set of strings over alphabet $\{a, b\}$ that contain at least one b.

(a) Draw a state-transition graph of a finite automaton that accepts $L_1 \cup L_2$. If such automaton does not exist, prove it.

Answer: See Figure 7.

(b) Draw a state-transition graph of a finite automaton that accepts $L_1 \cap L_2$. If such automaton does not exist, prove it.

Answer: See Figure 8.

(c) Write a complete formal definition of a context-free grammar that generates L_1L_2. If such grammar does not exist, prove it.

Answer: $G = \{V, \Sigma, P, S\}$, where
$\Sigma = \{a, b\}$, $V = \{S, L, R, A\}$, and the production set P is:

$$\begin{aligned}
&S \to LR \\
&L \to aAa \mid bAb \mid a \mid b \\
&R \to AbA \\
&A \to AA \mid \lambda \mid a \mid b
\end{aligned}$$

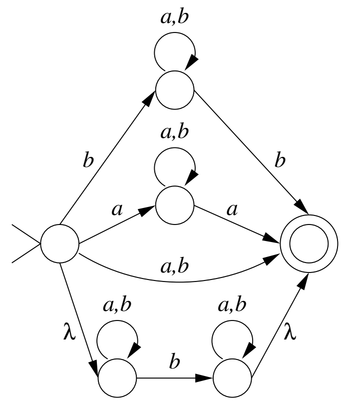

Figure 7:

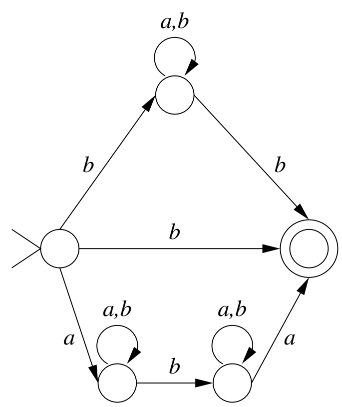

Figure 8:

Problem 4 Let L be the language defined by the regular expression:

$$(\boldsymbol{ca})^*(\boldsymbol{bd})^* \cup (\boldsymbol{aa})^*$$

(a) Construct a state-transition graph of a finite automaton that accepts L. If such an automaton does not exist, prove it.

Answer: See Figure 9.

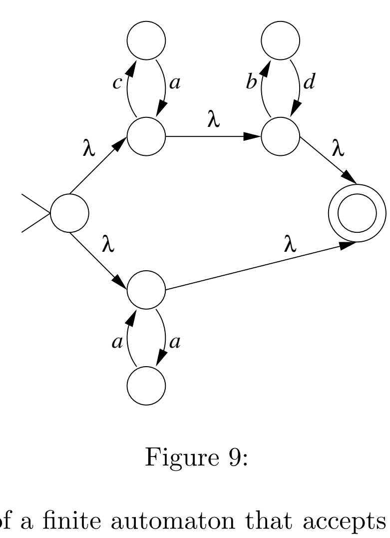

Figure 9:

(b) Construct a state-transition graph of a finite automaton that accepts L^*. If such an automaton does not exist, prove it.

Answer: See Figure 10.

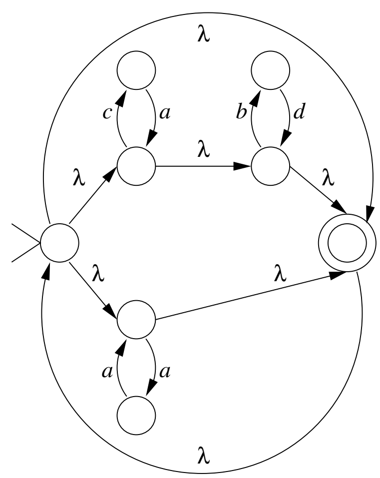

Figure 10: