CS320: Problems and Solutions for Day 10, Winter 2023

Problem 1 Let L be the set of strings over alphabet $\{a, b, c\}$ that do not start with c and do not end with a.

(a) Draw a state-transition graph of a deterministic finite automaton that accepts L. If such automaton does not exist, prove it.

Answer: See Figure 1.

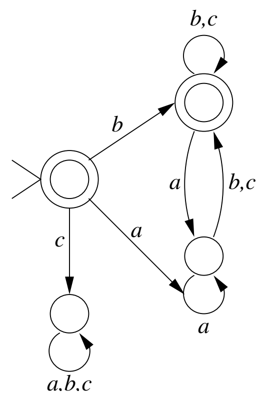

Figure 1:

(b) Is the complement $\overline{L}$ of the language L decidable? Explain your answer briefly.

Answer: Yes—L is regular, which is demonstrated by the finite automaton constructed in part (a); $\overline{L}$ is regular because L is regular and the class of regular languages is closed under complement; every regular language is decidable as the decision procedure consists of a simulation of the finite automaton that accepts the language.

Problem 2 Let L be the language accepted by the Turing machine:

$$M = (Q, \Sigma, \Gamma, \delta, q_0, F)$$

such that:
$Q = \{q_0, q_1, q_2, q_3, q_f\}$;
$\Sigma = \{a, b, c\}$;
$\Gamma = \{B, a, b, c\}$;
$F = \{q_f\}$;
and δ is defined by the following transition set:
$[q_0, a, q_1, a, R]$,
$[q_0, b, q_2, b, R]$,
$[q_0, c, q_3, c, R]$,
$[q_1, b, q_1, b, R]$,
$[q_2, c, q_2, c, R]$,
$[q_3, a, q_3, a, R]$,
$[q_1, B, q_f, B, R]$,
$[q_2, B, q_f, B, R]$,
$[q_3, B, q_f, B, R]$

Write a complete formal definition or a state-transition graph of a finite automaton M' that accepts L. If such automaton does not exist, prove it.

Advice for Answer: L is defined by the regular expression:

$$\boldsymbol{ab^* \cup bc^* \cup ca^*}$$

Problem 3 **(a)** Write a complete formal definition of a Turing machine M_1 over input alphabet $\{a, b\}$ such that M_1 **halts on every input.** If such a machine does not exist, explain why.

Answer:

$$M_1 = (Q, \Sigma, \Gamma, \delta, q_0)$$

where: $Q = \{q_0\}$; $\Sigma = \{a, b\}$; $\Gamma = \{B, a, b\}$; $\delta = \emptyset$.

(b) Write a complete formal definition of a Turing machine M_2 over input alphabet $\{a, b\}$ such that M_2 **does not halt on any input.** If such a machine does not exist, explain why.

Answer:

$$M_2 = (Q, \Sigma, \Gamma, \delta, q_0)$$

where: $Q = \{q_0\}$; $\Sigma = \{a, b\}$; $\Gamma = \{B, a, b\}$; and δ contains 3 transitions:

$$[q_0, a, q_0, a, R]\,,\ [q_0, b, q_0, b, R]\,,\ [q_0, B, q_0, B, R]$$

(c) Write a complete formal definition of a Turing machine M_3 over input alphabet $\{a, b\}$ such that M_3 **halts on every input and rejects Σ^*.** If such a machine does not exist, explain why.

Answer:

$$M_3 = (Q, \Sigma, \Gamma, \delta, q_0, F)$$

where:
$Q = \{q_0\}$; $\Sigma = \{a, b\}$; $\Gamma = \{B, a, b\}$; $\delta = \emptyset$; $F = \emptyset$

(d) Write a complete formal definition of a Turing machine M_4 over input alphabet $\{a, b\}$ such that M_4 **does not halt on any input and accepts Σ^*.** If such a machine does not exist, explain why.

Answer: Impossible. A machine cannot accept unless it halts.

Problem 4 Consider a Turing machine:

$$M = (Q, \Sigma, \Gamma, \delta, q)$$

such that:

$$Q = \{q_0, q_1\}$$
$$\Sigma = \{a, b\}$$
$$\Gamma = \{a, b, B\}$$

and δ is defined by the following transition set:

$$[q_0, a, q_1, B, R]$$
$$[q_0, b, q_1, B, R]$$
$$[q_1, a, q_0, B, R]$$
$$[q_1, b, q_0, B, R]$$
$$[q_1, B, q_1, B, R]$$

(where B is the designated blank symbol.)

(a) Write a complete formal definition of a Turing machine M_1 such that M_1 halts on input η if and only if M does not halt on input η, for all $\eta \in \Sigma^*$. In short:

$$(M(\eta) \searrow) \ \rightarrow \ (M_1(\eta) \nearrow)$$

and also:

$$(M(\eta) \nearrow) \ \rightarrow \ (M_1(\eta) \searrow)$$

If such Turing machine M_1 does not exist, prove it.

Answer: Observe that M halts on strings of even length. Hence, M_1 halts on strings of odd length.

$$M_1 = (Q, \Sigma, \Gamma, \delta_1, q)$$

such that:

$$Q = \{q_0, q_1\}$$
$$\Sigma = \{a, b\}$$
$$\Gamma = \{a, b, B\}$$

and δ_1 is defined by the following transition set:

$$[q_0, a, q_1, B, R]$$
$$[q_0, b, q_1, B, R]$$
$$[q_1, a, q_0, B, R]$$
$$[q_1, b, q_0, B, R]$$
$$[q_0, B, q_0, B, R]$$

(where B is the designated blank symbol.)

(b) Is the language accepted by M recursive? Explain your answer.

(c) Is the language accepted by M recursively enumerable? Explain your answer.

Problem 5 Consider the Turing machine:

$$M = (Q, \Sigma, \Gamma, \delta, q, F)$$

such that:
$Q = \{q, r, s, t, v\}$;
$\Sigma = \{a, b\}$;
$\Gamma = \{B, a, b, \Psi\}$;
$F = \{t\}$;
and δ is defined by the following transition set:

$$[q, a, q, a, R]$$
$$[q, b, q, \Psi, R]$$
$$[q, B, r, B, L]$$

$$[r, a, r, a, L]$$
$$[r, \Psi, s, \Psi, L]$$

$$[s, a, s, a, L]$$
$$[s, \Psi, t, \Psi, L]$$

$$[t, a, t, a, L]$$
$$[t, \Psi, v, \Psi, R]$$

$$[v, a, v, a, R]$$
$$[v, \Psi, v, \Psi, R]$$
$$[v, B, v, B, R]$$

(where B is the designated blank symbol.)
M accepts by final state.

(a) Write a regular expression that defines the set of strings on which M diverges. If such regular expression does not exist, prove it.

Answer:

$$\boldsymbol{a^*ba^*ba^*b(a \cup b)^*}$$

(b) Write a regular expression that defines the set of strings on which M halts and accepts. If such regular expression does not exist, prove it.

Answer:

$$\emptyset$$

(c) Write a regular expression that defines the set of strings on which M halts and rejects. If such regular expression does not exist, prove it.

Answer:

$$\emptyset$$

(d) Write a regular expression that defines the set of strings on which M terminates abnormally (attempts to move the head to the left of the leftmost cell.) If such regular expression does not exist, prove it.

Answer:

$$\boldsymbol{a^*(b \cup \lambda)a^*(b \cup \lambda)a^*}$$

Problem 6 Consider a Turing machine:

$$M = (Q, \Sigma, \Gamma, \delta, q_0)$$

such that:

$$Q = \{q_0, q_1, q_2\}$$
$$\Sigma = \{a, b, c\}$$
$$\Gamma = \{a, b, c, B\}$$

and δ is defined by the following transition set:

$$[q_0, a, q_1, a, R]$$
$$[q_0, b, q_0, b, R]$$
$$[q_0, c, q_0, c, R]$$
$$[q_0, B, q_0, B, R]$$
$$[q_1, a, q_2, a, R]$$
$$[q_1, b, q_1, b, R]$$
$$[q_1, c, q_1, c, R]$$
$$[q_1, B, q_1, B, R]$$

(where B is the designated blank symbol.)

(a) Let L be the set of those strings over Σ on which the Turing machine M does not halt. Draw a state transition graph of a deterministic finite automaton M_1 that accepts L. If such finite automaton M_1 does not exist, prove it.

Answer: See Figure 2.

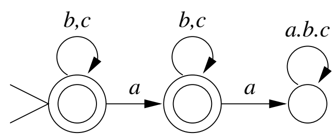

Figure 2:

(b) Is L a recursive language? Explain your answer briefly.

Answer: Yes—by the answer to part (a), L is regular. Every regular language is recursive, since a Turing machine can decide it by simulating a finite automaton that accepts it.

Problem 7 Consider the Turing machine:

$$M = (Q, \Sigma, \Gamma, \delta, q)$$

such that:
$Q = \{q, r, s, t\}$;
$\Sigma = \{a, b, c\}$;
$\Gamma = \{B, a, b, c\}$;
and δ is defined by the following transition set:

$$[q, a, r, b, R]$$
$$[q, b, r, a, R]$$
$$[q, c, t, c, R]$$
$$[t, a, t, a, R]$$
$$[t, b, t, b, R]$$
$$[t, B, s, B, R]$$

(where B is the designated blank symbol.)

(a) Does M halt on input *abba*? If your answer is "yes", write the configuration in which M halts. If your answer is "no", write the configuration of M after it makes exactly 9 moves.

Answer: Yes; M halts in the configuration:

$$brbba$$

(b) Does M halt on input *abcbbcb*? If your answer is "yes", write the configuration in which M halts. If your answer is "no", write the configuration of M after it makes exactly 9 moves.

Answer: Yes; M halts in the configuration:

$$brbcbbcb$$

(c) Does M halt on input *cbab*? If your answer is "yes", write the configuration in which M halts. If your answer is "no", write the configuration of M after it makes exactly 9 moves.

Answer: Yes; M halts in the configuration:

$$cbabBs$$