CS320: Problems for Days 9–13, Winter 2023

Problem 1 For each of the following six languages, give the *best possible* classification of that language into one of the following classes:

regular
context-free
recursive
recursively enumerable
not recursively enumerable

Your classification of a language is the best possible if the class which you indicate indeed contains that language, while no other listed class which is a proper subset of your class contains that language.

1. The language consisting of pairs $(R(M), w)$, where $R(M)$ is a representation of Turing machine M and M halts on input w.
2. The language consisting of representations of Turing machines that do not halt on blank tape.
3. $\emptyset$.
4. $\{0, 1, 111, 00\}$.
5. The language generated by the grammar:
   $G = \{\{S\}, \{a, b\}, P, S\}$, where P is given by:
   $S \to aS \mid bS \mid \lambda$.
6. The union of two arbitrary recursive languages.

Problem 2 **(a)** Give an example of a language that is not regular but has an infinite regular subset and an infinite regular superset. Give a precise definition of these three languages and explain your answer briefly. If such a language does not exist, explain why.

(b) Give an example of a language that is not context-free but has an infinite context-free subset and an infinite context-free superset. Give a precise definition of all these three languages and explain your answer briefly. If such a language does not exist, explain why.

(c) Give an example of a language that is not recursively enumerable but has a recursively enumerable complement. Give a precise definition of this language and explain your answer briefly. If such a language does not exist, explain why.

Problem 3 For each of the following claims, circle the word "**yes**" that follows the claim if the claim is correct, and circle the word "**no**" that follows the claim if the claim is not correct.

1. The union of any two regular languages is context-free.
2. The intersection of any two regular languages is regular.
3. The union of any two context-free languages is regular.
4. The intersection of any two context-free languages is context-free.
5. Every subset of a regular language is regular.
6. Every regular language is context-free.
7. Some context-free languages are not regular.
8. Some context-free languages are regular.
9. Every non-deterministic finite automaton has an equivalent deterministic finite automaton.
10. Some non-deterministic push-down automata have equivalent regular expressions.

11. Some finite automata have equivalent regular grammars.
12. Every finite automaton has an equivalent regular grammar.
13. Every regular grammar has an equivalent regular expression.
14. Every context-free grammar has an equivalent non-deterministic finite automaton.
15. Every regular expression has an equivalent deterministic finite automaton.
16. Every non-deterministic push-down automaton has an equivalent regular grammar.
17. Every subset of a context-free language is context-free.
18. The intersection of any regular language and any context-free language is context-free.
19. The intersection of any regular language and any context-free language is regular.
20. The union of any regular language and any context-free language is regular.
21. Every context-free language contains a regular subset.
22. The complement of every context-free language is context-free.
23. The complement of every regular language is context-free.
24. The complement of every regular language is regular.
25. Every recursively enumerable language is accepted by final state by some Turing machine.
26. Some recursively enumerable languages are not accepted by halting by any Turing machine.
27. Every recursive language is recursively enumerable.