# CS320: Problems for Day 9, Winter 2023

**Problem 1** Let $L$ be a language over alphabet $\Sigma = \{a, b, c, d, e\}$, defined as follows:

$$L = \{a^{m+1}b^{2n}c^{k+2}d^{3\ell}e^{j+3} \mid n = 2m, \ell = k + j\}$$

where $m, n, k, \ell, j \geq 0$.

**(a)** Write a complete formal definition of a context-free grammar $G$ that generates language $L$. If such grammar does not exist, prove it.

**(b)** Let $\mathcal{S}$ be a class of languages over alphabet $\{a, b, c, d, e\}$, defined as follows:

> Language $L$ is a member of $\mathcal{S}$ if and only if $L$ is generated by some context-free grammar, but there does not exist a pushdown automaton that accepts $L$.

What is the cardinality of $\mathcal{S}$? Explain your answer briefly.

**Problem 2** Let $L$ be the language accepted by the pushdown automaton:

$$M = (Q, \Sigma, \Gamma, \delta, q, F)$$

where:

$$Q = \{q, r, s, t\}$$
$$\Sigma = \{a, b\}$$
$$\Gamma = \{A\}$$
$$F = \{t\}$$

and the transition function $\delta$ is defined as follows:

$$[q, a, \lambda, r, A]$$
$$[r, a, \lambda, q, \lambda]$$
$$[t, b, \lambda, s, A]$$
$$[s, b, \lambda, t, \lambda]$$
$$[q, \lambda, \lambda, t, \lambda]$$

(Recall that $M$ is defined so as to accept by final state and empty stack.)

Write a regular expression that defines $L$. If such regular expression does not exist, prove it.

**Problem 3** Let $L_1$ be the language accepted by the pushdown automaton:

$$M = (Q, \Sigma, \Gamma, \delta, q, F)$$

where:

$$Q = \{q, r\}$$
$$\Sigma = \{a, b, c\}$$
$$\Gamma = \{A\}$$
$$F = \{r\}$$

and the transition function $\delta$ is defined as follows:

$$[q, a, \lambda, q, A]$$
$$[q, c, \lambda, r, \lambda]$$
$$[r, b, A, r, \lambda]$$

(Recall that $M$ is defined so as to accept by final state and empty stack.)

**(a)** Write a complete formal definition of a context-free grammar $G$ that generates $L_1$. If such grammar does not exist, prove it.

**(b)** Describe the algorithm that should be employed by a program that solves the following problem:

INPUT: An arbitrary string $x$ over $\Sigma$.

QUESTION: Does $x$ belong to $L_1$?

Explain your answer. If such algorithm does not exist, prove it.

**Problem 4** Let $L$ be the language accepted by the pushdown automaton:

$$M = (Q, \Sigma, \Gamma, \delta, q, F)$$

where:

$$\begin{aligned} Q &= \{q, r, s, t, v\} \\ \Sigma &= \{a, b, c, d\} \\ \Gamma &= \{A, B, D\} \\ F &= \{v\} \end{aligned}$$

and the transition function $\delta$ is defined as follows:

$$\begin{array}{l} [q, a, \lambda, q, A] \\ [r, b, A, r, \lambda] \\ [s, c, \lambda, s, B] \\ [t, a, B, t, \lambda] \\ [v, d, \lambda, v, D] \\ [v, d, D, v, \lambda] \\ [q, \lambda, \lambda, r, \lambda] \\ [r, \lambda, \lambda, s, \lambda] \\ [s, \lambda, \lambda, t, \lambda] \\ [t, \lambda, \lambda, v, \lambda] \end{array}$$

(Recall that $M$ is defined so as to accept by final state and empty stack.)

**(a)** Write a complete formal definition of a context-free grammar $G$ that generates $L$. If such grammar does not exist, prove it.

**(b)** Let $\mathcal{T}$ be a set of strings over alphabet $\{a, b, c\}$, defined as follows:

> String $w$ is a member of $\mathcal{T}$ if and only if $w$ is accepted by the pushdown automaton $M$ and the length of $w$ is greater than 6.

What is the cardinality of $\mathcal{T}$? Explain your answer briefly.

**Problem 5** Let $L$ be the language accepted by the pushdown automaton: $M = (Q, \Sigma, \Gamma, \delta, q, F)$, where

$$\begin{aligned} Q &= \{q, r\} \\ \Sigma &= \{a, b, c\} \\ \Gamma &= \{A, Z\} \\ F &= \{q\} \end{aligned}$$

and the transition function $\delta$ is defined as follows:

$$\begin{array}{l} [q, \lambda, \lambda, r, Z] \\ [q, \lambda, Z, q, \lambda] \\ [r, a, Z, r, ZA] \\ [r, c, A, r, \lambda] \\ [r, b, Z, q, \lambda] \end{array}$$

(Recall that $M$ is defined so as to accept by final state and empty stack. Furthermore, if an arbitrary stack string, say $X_1 \ldots X_n \in \Gamma^*$, where $n \geq 2$, is pushed on the stack by an individual transition, then the left-most symbol $X_1$ is pushed first, while the right-most symbol $X_n$ is pushed last.

**(a)** Write a regular expression that defines $L$. If such a regular expression does not exist, prove it.

**(b)** Write a complete formal definition of a context-free grammar $G$ that generates $L$. If such a grammar does not exist, prove it.

**Problem 6** Let $L$ be the language accepted by the pushdown automaton: $M = (Q, \Sigma, \Gamma, \delta, q, F)$, where $Q = \{q, r\}$, $\Sigma = \{a, b, c, d\}$, $\Gamma = \{A, B\}$, $F = \{r\}$, and the transition function $\delta$ is defined as follows:

$$\begin{array}{l} [q, a, \lambda, q, A] \\ [q, b, \lambda, q, B] \\ [q, c, \lambda, q, A] \\ [q, \lambda, \lambda, r, \lambda] \\ [r, d, A, r, \lambda] \\ [r, b, B, r, \lambda] \end{array}$$

(Recall that $M$ is defined so as to accept by final state and empty stack.)

**(a)** Write a complete formal definition of a context-free grammar $G$ that generates $L$. If such a grammar does not exist, prove it.

**(b)** Write a complete formal definition of a context-free grammar $G_1$ that generates $L^*$. If such a grammar does not exist, prove it.

**Problem 7** Let $L$ be the language accepted by the pushdown automaton: $M = (Q, \Sigma, \Gamma, \delta, q, F)$, where $Q = \{q, r, s\}$, $\Sigma = \{a, b, d\}$, $\Gamma = \{A\}$, $F = \{s\}$, and the transition function $\delta$ is defined as follows:

$$\begin{array}{l} [q, a, \lambda, q, A] \\ [r, d, \lambda, r, A] \\ [s, b, A, s, \lambda] \\ [q, \lambda, \lambda, r, \lambda] \\ [r, \lambda, \lambda, s, \lambda] \end{array}$$

(Recall that $M$ is defined so as to accept by final state and empty stack.)

**(a)** Write a complete formal definition of a context-free grammar $G$ that generates $L$. If such a grammar does not exist, prove it.

**(b)** Write a complete formal definition of a *regular* context-free grammar $G_1$ that generates $L$. If such a grammar does not exist, prove it.

**Problem 8** Let $L$ be the language accepted by the pushdown automaton:

$$M = (Q, \Sigma, \Gamma, \delta, q, F)$$

where:

$$\begin{array}{l} Q = \{q, r, s\} \\ \Sigma = \{a, b, c\} \\ \Gamma = \{A, B\} \\ F = \{s\} \end{array}$$

and the transition function $\delta$ is defined as follows:

$$\begin{array}{l} [q, a, \lambda, q, \lambda] \\ [q, b, \lambda, q, B] \\ [q, c, \lambda, q, \lambda] \\ [q, \lambda, \lambda, r, \lambda] \\ [r, \lambda, B, s, \lambda] \\ [s, \lambda, B, s, \lambda] \end{array}$$

(Recall that $M$ is defined so as to accept by final state and empty stack.)

Write a complete formal definition of a context-free grammar that generates $\overline{L}$ (the complement of $L$). If such a grammar does not exist, prove it.