CS320: Problems for Days 1–8, Winter 2023

Problem 1 Let:

$$\Sigma = \{a_1, a_2, \ldots, a_k\}$$
$$V = \{A_1, A_2, \ldots, A_m\}$$

and let:

$$N = \{0, 1, \ldots\}$$

be the set of natural numbers. State the cardinality of each of the following sets. (For a finite set, state the exact number. For an infinite set, specify if it is countable or not.)

Answer:

1. $\Sigma \times V$
2. V^*
3. $(\Sigma \times V)^*$
4. $\mathcal{P}(\Sigma)$
5. $\mathcal{P}(V^*)$
6. set of total functions from V to Σ
7. set of total functions from N to V
8. set of all context-free grammars of the form (V, Σ, P, S)
9. set of all regular expressions over Σ
10. language defined by the regular expression $\boldsymbol{(a_1 \cup a_2)(a_2 a_3)^*}$
11. $N \times \Sigma$
12. set of all languages over Σ

Problem 2 Let:

$$\Sigma = \{a, b, c, d, e\}$$
$$V = \{A, B, D, E, F, H, J, K\}$$

State the cardinality of each of the following sets. (For a finite set, state the exact number. For an infinite set, specify if it is countable or not.)

Answer:

1. $\Sigma \cup V$
2. $\Sigma^* \cup V^*$
3. $\Sigma \cap V$
4. $(\Sigma \cap V)^*$,
5. $\mathcal{P}(V)$
6. $\mathcal{P}(\Sigma^*)$
7. set of all regular expressions over Σ
8. set of all languages over Σ
9. set of total functions from Σ to V
10. set of all context-free grammars of the form (V, Σ, P, S)
11. $V \times \Sigma$

12. set of all languages over V

Problem 3 For each of the following claims, circle the word "**yes**" that follows the claim if the claim is correct, and circle the word "**no**" that follows the claim if the claim is not correct.

1. The class of context-free languages is closed under concatenation.
2. The Kleene star of any regular language is regular.
3. The Kleene star of any regular language is context-free.
4. The union of any two context-free languages is context-free.
5. Every finite language is regular.
6. There exists an algorithm to convert any regular expression into an equivalent finite automaton.
7. There exists an algorithm to convert any regular expression into an equivalent context-free grammar.
8. Every deterministic finite automaton is equivalent to some non-deterministic finite automaton.
9. Every non-deterministic finite automaton is equivalent to some deterministic finite automaton.
10. Every context-free grammar is equivalent to some regular expression.
11. Set of strings of the form $\{a^k b^k c^k \mid k \geq 0\}$ is regular.
12. Set of strings of the form $\{a^k b^k c^k \mid k \geq 0\}$ is context-free.
13. Set of strings of the form $\{a^k b^k \mid k \geq 0\}$ is regular.
14. Set of strings of the form $\{a^k b^k \mid k \geq 0\}$ is context-free.
15. Every language has a finite description.
16. Every finite language has a finite description.
17. Set $\{a, b\}^*$ is regular.
18. Every subset of $\{a, b\}^*$ is regular.
19. Every subset of $\{a, b\}^*$ is context-free.

Problem 4 For each of the following claims, circle the word "**yes**" that follows the claim if the claim is correct, and circle the word "**no**" that follows the claim if the claim is not correct.

1. The concatenation of any two regular languages is context-free.
2. The Kleene star of any regular language is context-free.
3. Every subset of a regular language is regular.
4. Some context-free languages are not regular.
5. Some finite languages are not regular.
6. Some regular languages are not finite.
7. The union of any two regular languages is regular.
8. Language $\{a^n b^m c^k \mid k = m + n,\ k, m, n \geq 0\}$
   is context-free.
9. Language $\{a^n b^m c^k \mid k, m, n \geq 0\}$ is not regular.
10. Language $\{a^n b^m c^k d^\ell \mid n = k \wedge m = \ell,\ k, m, n, \ell \geq 0\}$
    is context-free.

11. Language $\{a^n b^m c^k d^\ell \mid n = k \vee m = \ell,\ k, m, n, \ell \geq 0\}$
is context-free.
12. Every regular language has a context-free subset.
13. Every infinite context-free language has an infinite regular superset.
14. Language $\{a^n b^m c^k d^\ell \mid n = k \wedge m = \ell,\ k, m, n, \ell \geq 0\}$
has a proper subset which is context-free and infinite.
15. Every infinite language has a finite description.
16. Every finite language is generated by some context-free grammar.
17. Some infinite languages cannot be described by a regular expression.
18. Set $\{a, b\}^*$ has a proper subset which is infinite and not context-free.
19. The intersection of $\{a^n b^m c^m \mid m, n \geq 0\}$ and $\{a^n b^n c^m \mid m, n \geq 0\}$ is context-free.
20. The intersection of $\{a^n b^m c^m \mid m, n \geq 0\}$ and $\{a^n b^n c^m \mid m, n \geq 0\}$ has a context-free complement.

Problem 5 Fill the empty box at the end of each of the numbered claims with one of these signs:

$$-,\ \surd,\ ?$$

so that the meaning of the signs is as follows:

- $-$ means that the preceding claim is always false;
- $\surd$ means that the preceding claim is always true;
- ? means that the preceding claim may be true but may be false, depending on the values of the variable(s) appearing in the claim.

Problem assumptions: L_1 is an arbitrary regular language; L_2 is an arbitrary context-free language; e_1 is a regular expression that represents L_1; G_2 is a context-free grammar that generates L_2.

Claims:

1. L_1 is context-free.
2. L_2 is regular.
3. L_2^* is context-free.
4. L_1 has a regular complement.
5. $L_1 \cap L_2$ is regular.
6. $L_1 \cap L_2$ is context-free.
7. L_2 has a context-free complement.
8. $L_1 L_2$ is context-free.
9. There exists an algorithm which on input e_1 outputs a regular context-free grammar for L_1.
10. There exists an algorithm which on input G_2 outputs a regular context-free grammar for L_2.