CS320: Problems for Day 6, Winter 2023

Problem 1 Let L_1 be the set of strings over alphabet $\{a, b, c\}$ in which the total number of b's and c's is one. Let L_2 be the set of strings over alphabet $\{a, b, c\}$ that contain an even number of b's.

(a) Draw a state-transition graph of a finite automaton M_1 that accepts $L_1 \cup L_2$. If such automaton does not exist, prove it.

(b) Draw a state-transition graph of a finite automaton M_2 that accepts $(L_1 L_2)^*$. If such automaton does not exist, prove it.

(c) Is $L_1 \cap L_2$ a context-free language? Explain your answer.

Problem 2 Let L_1 be the set of strings over alphabet $\{a, b, c\}$ that contain at least one b. Let L_2 be the set of strings over alphabet $\{a, b, c\}$ whose length gives remainder 2 when divided by 3.

(a) Draw a state-transition graph of a finite automaton that accepts L_1. If such automaton does not exist, prove it.

(b) Draw a state-transition graph of a finite automaton that accepts L_2. If such automaton does not exist, prove it.

(c) Draw a state-transition graph of a finite automaton that accepts $L_1 \cup L_2$. If such automaton does not exist, prove it.

(d) Draw a state-transition graph of a finite automaton that accepts $L_1 \cap L_2$. If such automaton does not exist, prove it.

Problem 3 Let L_1 be the set of all strings over alphabet $\{a, b\}$ whose first and last letters are equal. Let L_2 be the set of strings over alphabet $\{a, b\}$ that contain at least one b.

(a) Draw a state-transition graph of a finite automaton that accepts $L_1 \cup L_2$. If such automaton does not exist, prove it.

(b) Draw a state-transition graph of a finite automaton that accepts $L_1 \cap L_2$. If such automaton does not exist, prove it.

(c) Write a complete formal definition of a context-free grammar that generates $L_1 L_2$. If such grammar does not exist, prove it.

Problem 4 Let L be the language defined by the regular expression:

$$(\boldsymbol{ca})^*(\boldsymbol{bd})^* \cup (\boldsymbol{aa})^*$$

(a) Construct a state-transition graph of a finite automaton that accepts L. If such an automaton does not exist, prove it.

(b) Construct a state-transition graph of a finite automaton that accepts L^*. If such an automaton does not exist, prove it.