CS320: Problems for Day 3, Winter 2023

Problem 1 Let $N = \{0, 1, \ldots\}$ be the set of natural numbers and let L be the language defined by the regular expression:

$$(\boldsymbol{a} \cup \boldsymbol{\lambda})\,(\boldsymbol{a} \cup \boldsymbol{\lambda})\,\boldsymbol{b}^*$$

Construct a bijection from L to N. If such bijection does not exist, prove it.

Problem 2 Let:

$$\Sigma = \{0, 1\}$$

and let L be the language defined by the regular expression:

$$(\mathbf{0} \cup \mathbf{1})\,\mathbf{01}$$

Let N be the set of natural numbers.

State the cardinality of each of the following sets. (For a finite set, state the exact number. For an infinite set, specify if it is countable or not.)

1. L
2. L^*
3. L^0
4. set of all strings of length 5 over Σ
5. set of all strings of finite length over Σ
6. set of subsets of Σ
7. set of subsets of Σ^*
8. set of all languages over Σ
9. set of all functions from N to Σ
10. set of regular expressions over Σ

Problem 3 **(a)** Give an example of a finite language that is not regular. Give a precise definition of this language and explain your answer briefly. If such a language does not exist, explain why.

(b) Give an example of a regular language that is not finite. Give a precise definition of this language and explain your answer briefly. If such a language does not exist, explain why.

(c) Give an example of an infinite uncountable language. Give a precise definition of this language and explain your answer briefly. If such a language does not exist, explain why.

Problem 4 Let L be the language defined by the regular expression:

$$(\boldsymbol{c} \cup \boldsymbol{ab} \cup \boldsymbol{b})^*\boldsymbol{bb}\,(\boldsymbol{c} \cup \boldsymbol{ab})$$

(a) Write 10 distinct strings that belong to L. If such strings do not exist, explain why.

(b) Write 10 distinct strings over alphabet $\{a, b, c\}$ that do not belong to L. If such strings do not exist, explain why.

Problem 5 Let L be a set of strings over alphabet $\{0, 1\}$ such that

$$L = \{w \mid w \neq 11 \wedge w \neq 111\}$$

(a) Write 5 distinct strings that belong to L. If such strings do not exist, explain why.

(b) Write 5 distinct strings over alphabet $\{0, 1\}$ that do not belong to L. If such strings do not exist, explain why.

(c) Write a regular expression that defines L. If such expression does not exist, explain why.

Problem 6 Let L be the language defined by the regular expression:

$$(\boldsymbol{a} \cup \boldsymbol{b})(\boldsymbol{a} \cup \boldsymbol{b})(\boldsymbol{a} \cup \boldsymbol{b})^*$$

(a) Write 8 distinct strings that belong to L. If such strings do not exist, explain why.

(b) Write 8 distinct strings over alphabet $\{a, b\}$ that do not belong to L. If such strings do not exist, explain why.

Problem 7 Let L_1 be the language defined by the regular expression:

$$(\boldsymbol{a} \cup \boldsymbol{b})\ (\ (\boldsymbol{a} \cup \boldsymbol{b})(\boldsymbol{a} \cup \boldsymbol{b})\ )^*$$

and let L_2 be the language defined by the regular expression:

$$(\boldsymbol{a} \cup \boldsymbol{b})\ (\boldsymbol{a} \cup \boldsymbol{b})\ (\boldsymbol{a} \cup \boldsymbol{b})^*$$

In your answers to the following questions, state cardinalities of finite sets by giving the exact numbers. For infinite sets, specify if they are countable or not.

(a) Let $S_1 = L_1 \cup L_2$. Write a regular expression that defines S_1. If such a regular expression does not exist, explain why.

(b) Let $S_2 = L_1 \setminus L_2$. Write a regular expression that defines S_2. If such a regular expression does not exist, explain why.

(c) Let $S_3 = L_1 \cap L_2$. Write a regular expression that defines S_3. If such a regular expression does not exist, explain why.

(d) What is the cardinality of S_1?

(e) What is the cardinality of S_2?

(f) What is the cardinality of S_3?

Problem 8 **(a)** Let L_1 be the set of all strings of length two or more over alphabet $\{a, b\}$ in which all the a's follow all the b's. Write 5 distinct strings that belong to L_1. If such strings do not exist, explain why.

(b) Write a regular expression that defines L_1. If such a regular expression does not exist, explain why.

(c) Let L_2 be the set of all strings over alphabet $\{a, b, c\}$ that begin with c, end with b, and contain exactly two a's. Write 5 distinct strings that belong to L_2. If such strings do not exist, explain why.

(d) Write a regular expression that defines L_2. If such a regular expression does not exist, explain why.

Problem 9 **(a)** Let L_1 be the set of all strings over alphabet $\{0, 1\}$ that do not contain the substring 11.

Write a regular expression that defines L_1. If such regular expression does not exist, explain why.

(b) Let L_2 be the set of all strings over alphabet $\{0, 1\}$ that contain an odd number of 1's.

Write a regular expression that defines L_2. If such regular expression does not exist, explain why.

Problem 10 **(a)** Let L_1 be the set of all strings over alphabet $\{0, 1\}$ that contain exactly two zeros.

Write a regular expression that defines L_1. If such regular expression does not exist, explain why.

(b) Let L_2 be the set of all strings over alphabet $\{0, 1\}$ that contain at least two zeros.

Write a regular expression that defines L_2. If such regular expression does not exist, explain why.

(c) Let L_3 be the set of all strings over alphabet $\{0, 1\}$ that contain at most two zeros.

Write a regular expression that defines L_3. If such regular expression does not exist, explain why.

Problem 11 **(a)** Let L be the set of strings over the alphabet $\{0, 1\}$ whose first symbol is different from the last symbol. Write a regular expression that defines L. If such a regular expression does not exist, prove it.

(b) Let $\mathcal{R}$ be the class of languages that can be represented by a regular expression, and let $\mathcal{N}$ be the class of languages that cannot be represented by a regular expression. State the cardinalities of $\mathcal{R}$ and $\mathcal{N}$, and compare them.

Problem 12 Let L be the set of all strings over alphabet $\{a, b, c\}$ whose first letter occurs at least once again in the string.

Write a regular expression that defines L. If such a regular expression does not exist, prove it.