CS320: Problems for Day 2, Winter 2023

Problem 1 Let N be the set of natural numbers and let $\mathcal{F}$ be the set of functions from the set N to the set $\{0, 1\}$.
(a) Construct an injective function $g_1 : N \to \mathcal{F}$. If such a function does not exist, explain why.
(b) Construct an injective function $g_2 : \mathcal{F} \to N$. If such a function does not exist, explain why.

Problem 2 Let N be the set of natural numbers and let $\mathcal{F}$ be the set of functions from set N to set $\{0, 1\}$.
(a) Construct an injective function:

$$g_1 : N \to \mathcal{F}$$

If such function does not exist, explain why.
(b) Construct a surjective function:

$$g_2 : N \to \mathcal{P}(N)$$

If such function does not exist, explain why.
(c) Construct a bijective function:

$$g_3 : \mathcal{P}(N) \to \mathcal{F}$$

If such function does not exist, explain why.

Problem 3 Let:

$$\Sigma = \{a, b, c\}$$

and let:

$$N = \{0, 1, \ldots\}$$

be the set of natural numbers. State the cardinality of each of the following sets. (For a finite set, state the exact number. For an infinite set, specify if it is countable or not.)

1. $\mathcal{P}(\Sigma)$
2. Σ^*
3. set of functions from Σ to $\{0, 1\}$
4. set of functions from Σ^* to $\{0, 1\}$
5. $\mathcal{P}(\Sigma^*)$
6. set of functions from N to Σ
7. $\Sigma \times \Sigma$
8. $\Sigma^* \times \Sigma^*$
9. set of all finite subsets of N
10. $\mathcal{P}(N)$

Problem 4 **(a)** Give an example of two countably infinite sets, S_1 and S_2, such that $S_1 \cap S_2$ is finite. (Define S_1 and S_2 precisely.) If such sets do not exist, explain why.
(b) Give an example of two countably infinite sets, S_1 and S_2, such that $S_1 \setminus S_2$ is infinite. (Define S_1 and S_2 precisely.) If such sets do not exist, explain why.

Problem 5 Let $N = \{0, 1, \ldots\}$ be the set of natural numbers. Construct five different injective functions from N to $N \times N$. If such functions do not exist, explain why.

Problem 6 Let $N = \{0, 1, \ldots\}$ be the set of natural numbers.
(a) Construct an injective function from $N \times N$ to N. Justify your answer briefly. If such a function does not exist, explain why.
(b) Construct a non-injective function from $N \times N$ to N. Justify your answer briefly. If such a function does not exist, explain why.

Problem 7 **(a)** Construct an injective function from $N \times N$ to $N \times N \times N$. Justify your answer briefly. If such a function does not exist, explain why.

(b) Construct an injective function from $N \times N \times N$ to $N \times N$. Justify your answer briefly. If such a function does not exist, explain why.

(c) Construct a surjective function from $N \times N \times N$ to $N \times N$. Justify your answer briefly. If such a function does not exist, explain why.