# CS320: Problems for Day 11, Winter 2023

**Problem 1** You are given two Turing machines, $M_1$ and $M_2$, such that $M_1$ *accepts* language $L_1$ and $M_2$ *decides* language $L_2$.

Is $L_1 \setminus L_2$ a recursively enumerable language?
If your answer is "yes", prove it by describing an appropriate Turing machine. If your answer is "no", prove it by showing that such a Turing machine does not exist.

**Problem 2** You are given two Turing machines, $M_1$ and $M_2$, such that $M_1$ *accepts* language $L_1$ and $M_2$ *accepts* language $L_2$.

Is $L_1 \cup L_2$ a recursively enumerable language?
If your answer is "yes", prove it by describing an appropriate Turing machine. If your answer is "no", prove it by showing why such a Turing machine does not exist.

**Problem 3** Let:

$$L = \{(R(M), n) \mid M \text{ halts on blank tape after } \leq n \text{ steps } \}$$

where $R(M)$ is a representation of Turing machine $M$ and $n$ is a natural number. Describe a Turing machine $M'$ that accepts $L$. If such $M'$ does not exist, explain why.

**Problem 4** Let $L$ be a non-recursive language, accepted by a Turing machine $M$, and let $k$ be a natural number. Describe a Turing machine $M'$, such that on input $w$, $M'$ writes *error* on its tape and halts if and only if $M$ does not accept $w$ within the first $k$ computation steps. If such $M'$ does not exist, explain why.