# CS320: Problems for Day 10, Winter 2023

**Problem 1** Let $L$ be the set of strings over alphabet $\{a, b, c\}$ that do not start with $c$ and do not end with $a$.

**(a)** Draw a state-transition graph of a deterministic finite automaton that accepts $L$. If such automaton does not exist, prove it.

**(b)** Is the complement $\overline{L}$ of the language $L$ decidable? Explain your answer briefly.

**Problem 2** Let $L$ be the language accepted by the Turing machine:

$$M = (Q, \Sigma, \Gamma, \delta, q_0, F)$$

such that:
$Q = \{q_0, q_1, q_2, q_3, q_f\}$;
$\Sigma = \{a, b, c\}$;
$\Gamma = \{B, a, b, c\}$;
$F = \{q_f\}$;
and $\delta$ is defined by the following transition set:
$[q_0, a, q_1, a, R]$,
$[q_0, b, q_2, b, R]$,
$[q_0, c, q_3, c, R]$,
$[q_1, b, q_1, b, R]$,
$[q_2, c, q_2, c, R]$,
$[q_3, a, q_3, a, R]$,
$[q_1, B, q_f, B, R]$,
$[q_2, B, q_f, B, R]$,
$[q_3, B, q_f, B, R]$

Write a complete formal definition or a state-transition graph of a finite automaton $M'$ that accepts $L$. If such automaton does not exist, prove it.

**Problem 3** **(a)** Write a complete formal definition of a Turing machine $M_1$ over input alphabet $\{a, b\}$ such that $M_1$ **halts on every input.** If such a machine does not exist, explain why.

**(b)** Write a complete formal definition of a Turing machine $M_2$ over input alphabet $\{a, b\}$ such that $M_2$ **does not halt on any input.** If such a machine does not exist, explain why.

**(c)** Write a complete formal definition of a Turing machine $M_3$ over input alphabet $\{a, b\}$ such that $M_3$ **halts on every input and rejects $\Sigma^*$.** If such a machine does not exist, explain why.

**(d)** Write a complete formal definition of a Turing machine $M_4$ over input alphabet $\{a, b\}$ such that $M_4$ **does not halt on any input and accepts $\Sigma^*$.** If such a machine does not exist, explain why.

**Problem 4** Consider a Turing machine:

$$M = (Q, \Sigma, \Gamma, \delta, q)$$

such that:

$$Q = \{q_0, q_1\}$$
$$\Sigma = \{a, b\}$$
$$\Gamma = \{a, b, B\}$$

and $\delta$ is defined by the following transition set:

$$[q_0, a, q_1, B, R]$$
$$[q_0, b, q_1, B, R]$$
$$[q_1, a, q_0, B, R]$$
$$[q_1, b, q_0, B, R]$$
$$[q_1, B, q_1, B, R]$$

(where $B$ is the designated blank symbol.)

**(a)** Write a complete formal definition of a Turing machine $M_1$ such that $M_1$ halts on input $\eta$ if and only if $M$ does not halt on input $\eta$, for all $\eta \in \Sigma^*$. In short:

$$(M(\eta) \searrow) \;\rightarrow\; (M_1(\eta) \nearrow)$$

and also:

$$(M(\eta)\nearrow) \;\rightarrow\; (M_1(\eta)\searrow)$$

If such Turing machine $M_1$ does not exist, prove it.

**(b)** Is the language accepted by $M$ recursive? Explain your answer.

**(c)** Is the language accepted by $M$ recursively enumerable? Explain your answer.

**Problem 5** Consider the Turing machine:

$$M = (Q, \Sigma, \Gamma, \delta, q, F)$$

such that:
$Q = \{q, r, s, t, v\}$;
$\Sigma = \{a, b\}$;
$\Gamma = \{B, a, b, \Psi\}$;
$F = \{t\}$;
and $\delta$ is defined by the following transition set:

$$\begin{array}{l}
[q, a, q, a, R] \\
[q, b, q, \Psi, R] \\
[q, B, r, B, L] \\
\\
[r, a, r, a, L] \\
[r, \Psi, s, \Psi, L] \\
\\
[s, a, s, a, L] \\
[s, \Psi, t, \Psi, L] \\
\\
[t, a, t, a, L] \\
[t, \Psi, v, \Psi, R] \\
\\
[v, a, v, a, R] \\
[v, \Psi, v, \Psi, R] \\
[v, B, v, B, R]
\end{array}$$

(where $B$ is the designated blank symbol.)
$M$ accepts by final state.

**(a)** Write a regular expression that defines the set of strings on which $M$ diverges. If such regular expression does not exist, prove it.

**(b)** Write a regular expression that defines the set of strings on which $M$ halts and accepts. If such regular expression does not exist, prove it.

**(c)** Write a regular expression that defines the set of strings on which $M$ halts and rejects. If such regular expression does not exist, prove it.

**(d)** Write a regular expression that defines the set of strings on which $M$ terminates abnormally (attempts to move the head to the left of the leftmost cell.) If such regular expression does not exist, prove it.

**Problem 6** Consider a Turing machine:

$$M = (Q, \Sigma, \Gamma, \delta, q_0)$$

such that:

$$\begin{array}{l}
Q = \{q_0, q_1, q_2\} \\
\Sigma = \{a, b, c\} \\
\Gamma = \{a, b, c, B\}
\end{array}$$

and $\delta$ is defined by the following transition set:

$$\begin{array}{l} [q_0, a, q_1, a, R] \\ [q_0, b, q_0, b, R] \\ [q_0, c, q_0, c, R] \\ [q_0, B, q_0, B, R] \\ [q_1, a, q_2, a, R] \\ [q_1, b, q_1, b, R] \\ [q_1, c, q_1, c, R] \\ [q_1, B, q_1, B, R] \end{array}$$

(where $B$ is the designated blank symbol.)

**(a)** Let $L$ be the set of those strings over $\Sigma$ on which the Turing machine $M$ does not halt. Draw a state transition graph of a deterministic finite automaton $M_1$ that accepts $L$. If such finite automaton $M_1$ does not exist, prove it.

**(b)** Is $L$ a recursive language? Explain your answer briefly.

**Problem 7** Consider the Turing machine:

$$M = (Q, \Sigma, \Gamma, \delta, q)$$

such that:
$Q = \{q, r, s, t\}$;
$\Sigma = \{a, b, c\}$;
$\Gamma = \{B, a, b, c\}$;
and $\delta$ is defined by the following transition set:

$$\begin{array}{l} [q, a, r, b, R] \\ [q, b, r, a, R] \\ [q, c, t, c, R] \\ [t, a, t, a, R] \\ [t, b, t, b, R] \\ [t, B, s, B, R] \end{array}$$

(where $B$ is the designated blank symbol.)

**(a)** Does $M$ halt on input *abba*? If your answer is "yes", write the configuration in which $M$ halts. If your answer is "no", write the configuration of $M$ after it makes exactly 9 moves.

**(b)** Does $M$ halt on input *abcbbcb*? If your answer is "yes", write the configuration in which $M$ halts. If your answer is "no", write the configuration of $M$ after it makes exactly 9 moves.

**(c)** Does $M$ halt on input *cbab*? If your answer is "yes", write the configuration in which $M$ halts. If your answer is "no", write the configuration of $M$ after it makes exactly 9 moves.