QUEENS COLLEGE   Department of Computer Science
CSCI 320   Final Exam   Winter 2023   1.24.23

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10.00am – 11.30am, Tuesday, January 24, 2023

Problem 1 For each of the following five languages, give the *best possible* classification of that language into one of the following classes:

regular
context-free
decidable
Turing recognizable
not Turing recognizable

Your classification of a language is the best possible if the class which you indicate contains that language, but no other listed class which is a proper subset of your class contains that language.

1. The language consisting of strings of the form $a^n a^n c^n$ where n is a positive integer.

   Answer: context-free

2. The language consisting of strings of the form $a^n b^n a^n$ where n is a positive integer.

   Answer: decidable

3. The language consisting of all binary strings.

   Answer: regular

4. The language consisting of pairs (M, w), where M is (the binary code for) a Turing machine that diverges on input w.

   Answer: not Turing recognizable

5. The language consisting of pairs (M, w), where M is (the binary code for) a Turing machine that halts and rejects on input w.

   Answer: Turing recognizable

Problem 2 Let L be the language accepted by the pushdown automaton:

$$M = (Q, \Sigma, \Gamma, \delta, q, F)$$

where:

$$Q = \{q, r, s, t\} \quad \Sigma = \{a, b, c\} \quad \Gamma = \{\$\} \quad F = \{q\}$$

and the transition function δ is defined by the following state diagram:

(Recall that M is defined so as to accept by final state and empty stack.)

(a) Write 3 distinct strings that belong to L. If such strings do not exist, prove it.

Answer: λ, cc, acc

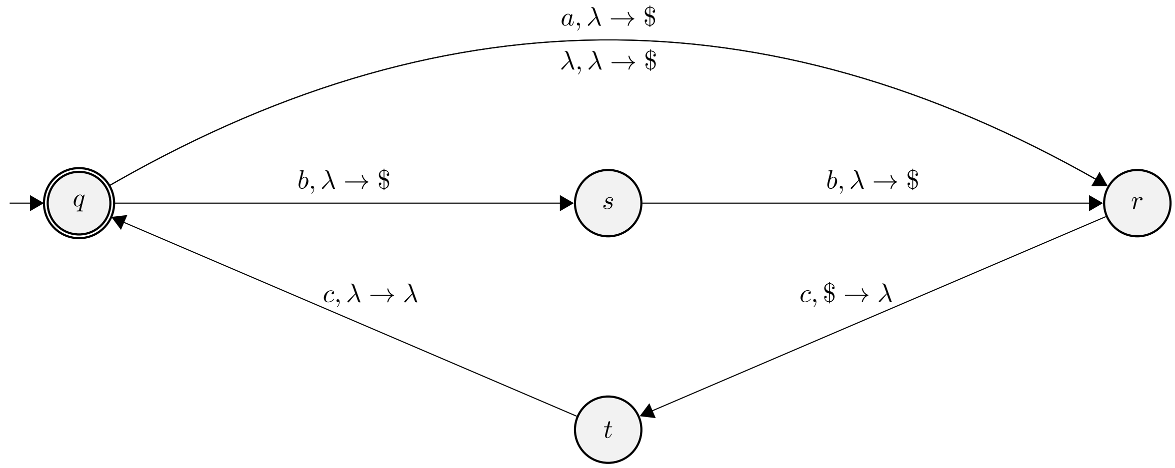
(b) Give a context free grammar G that generates L. If such a grammar does not exist, prove it.

Answer: $G = (V, \Sigma, P, S)$, where $\Sigma = \{a, c\}$, $V = \{S, L\}$, and the production set P is:

$$S \to LS \mid \lambda$$
$$L \to acc \mid cc$$

(c) Give a regular expression that represents L. If such an expression does not exist, prove it.

Answer: $(cc \cup acc)^*$

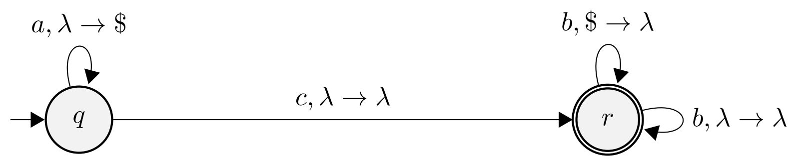


Problem 3 Let L be the language generated by the context-free grammar $G = (V, \Sigma, P, S)$, where: $\Sigma = \{a, b, c\}$, $V = \{S\}$, and P is:

$$S \to aSb \;|Sb \mid c$$

(a) Construct the state diagram of a push down automaton that accepts the language L. If such a push down automaton does not exist, prove it.

Answer:



(b) Draw a state-transition graph of a deterministic finite automaton that accepts the language L. If such an automaton does not exist, prove it.

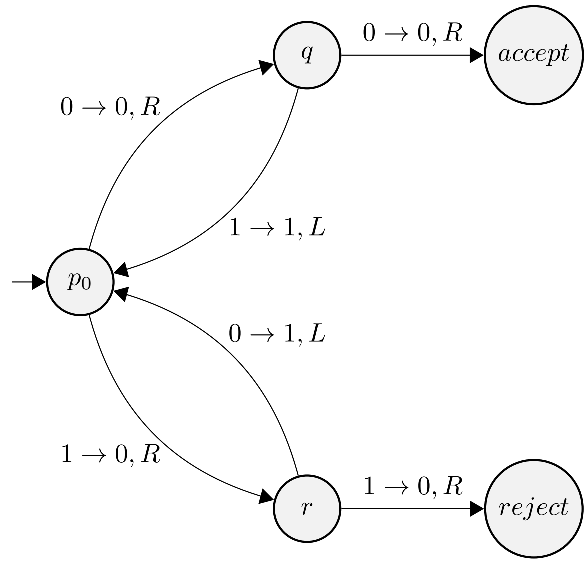
Answer: Such an expression does not exist, since L is not a regular language. To prove this, assume the opposite. Let k be the pumping constant for L. Then $a^k cb^k \in L$. In the "pumping" decomposition: $a^k cb^k = uvx$, we have: $|uv| \le k$, hence the "pumping" substring v consists entirely of a's, say $v = a^j$. Recall that $j > 0$, since the "pumping" substring cannot be empty. By the pumping lemma, the word $W = uv^2x$ belongs to L. However, w has the form $a^{k+j}cb^k$. The word w does not belong to L, since $\$^j$ remains on the stack when w is passed through the push down automaton of (a). This is a contradiction.

Problem 4 Consider the Turing machine:

$$M = (Q, \Sigma, \Gamma, \delta, p_0, accept, reject)$$

where: $Q = \{p_0, q, r, accept, reject\}$; $\Sigma = \{0, 1\}$; $\Gamma = \{B, 0, 1\}$;

Here, M has a one-way infinite tape, B is the designated blank symbol, p_0 is the inital state, and *accept* and *reject* are the accepting and rejecting states. The transition rule δ is defined by the following diagram:



In answers to the following questions, a list of configurations should begin with the initial configuration of the machine.

(a) Write the first three configurations of the machine as it processes the input string $w = 00$. Does M halt on input string w?

Answer: $p_0 00$, $0q0$, $00accept$. M halts when it reaches the accept state.

(b) Write the first three configurations of the machine as it processes the input string $w = 1100$. Does M halt on input string w?

Answer: $p_0 1100$, $0r100$, $00reject00$. M halts when it reaches the reject state.

(c) Write the first three configurations of the machine as it processes the input string $w = 01$. Does M halt on input string w?

Answer: $p_0 01$, $0q1$, $p_0 01$. M does not halt on input string w because its third configuration is identical to its first. This starts an infinite loop.

(d) Write the first five configurations of the machine as it processes the input string $w = 100$. Does M halt on input string w?

Answer: $p_0 100$, $0r00$, $p_0 010$, $0q10$, $p_0 010$. M does not halt on input string w because its fifth configuration is identical to its third. This starts an infinite loop.

(e) Write a regular expression that defines the set of strings that are accepted by M. If such a regular expression does not exist, prove it.

Answer: $00(0 \cup 1)^*$, because a string is accepted if and only of it begins 00.

Problem 5 Let L be the set of strings that have the form ww where w is a string in $(a \cup b \cup c)^*$.

Let L_1 be the language defined by the regular expression: $\boldsymbol{a^*b^*c^*}$

Let L_2 be the language defined by the regular expression: $\boldsymbol{a^*b^*c^*a^*b^*c^*}$

(a) Let $S_1 = L \cap L_1$. Write a regular expression that defines S_1. If such a regular expression does not exist, prove it.

Answer: $(aa)^* \cup (bb)^* \cup (cc)^*$

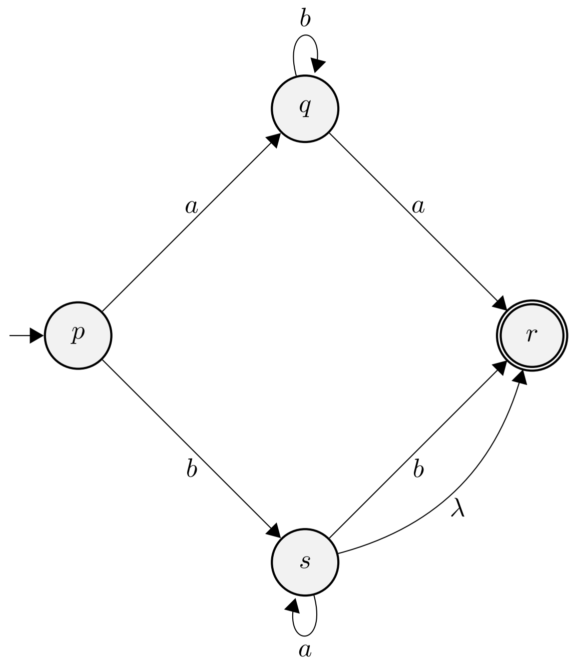
(b) Let $S_2 = L \cap L_2$. Write a context free grammar that defines S_2. If such a grammar does not exist, prove it.

Answer: Such a context free grammar does not exist.

S_2 consists of strings with the form $a^\ell b^m c^n a^\ell b^m c^n$, where $\ell \geq 0, m \geq 0, n \geq 0$.

Suppose that S_2 is context free. Then it has a pumping constant k. Consider the string $w = a^k b^k c^k a^k b^k c^k \in S_2$. We can pump w, because $\ell(w) = 6k \geq k$. In a pumping decomposition $w = sxyzt$. The pumping window xyz has $\ell(xyz) \leq k$. Moreover, the pumping part xz is non-empty. Suppose that $\alpha \in \{a, b, c\}$ appears exactly $j \geq 1$ times in xz. Note that the pumping window can only meet one of the groups of α characters in w — because they are separated by $2k$ other characters and xyz has length at most k. Hence, if the pumped string $w' = sxxyzzt$ has only two groups of α characters, one has length $k+j$ and the other has length k. We deduce that $w' \notin S_2$, in contradiction to the Pumping Lemma for context free languages. The contradiction shows that we were wrong to suppose that S_2 is context free.

Problem 6 Let L be the language accepted by the NFA with the following state transition graph.



Draw the state-transition graph of the deterministic finite automaton that accepts L that is constructed by our algorithm that converts an NFA to a DFA. If such an automaton does not exist, prove it.

Answer: