QUEENS COLLEGE Department of Computer Science
CSCI 320 Midterm Exam Winter 2023 1.13.23
Instructor: Alex Ryba

10.00am – 11.30am, Friday, January 13, 2023

Problem 1 **(a)** Give an example of a finite language that is not regular. Give a precise definition of this language and explain your answer briefly. If such a language does not exist, explain why.

Answer: Such a language does not exist. Every finite language is regular.

(b) Give an example of a regular language that is not finite. Give a precise definition of this language and explain your answer briefly. If such a language does not exist, explain why.

Answer: Let $\Sigma = \{0, 1\}$. Then Σ^* is regular and is not finite.

(c) Give an example of an infinite uncountable language. Give a precise definition of this language and explain your answer briefly. If such a language does not exist, explain why.

Answer: There is no such language.

Consider a language L over an alphabet Σ. Then Σ^* is countable, but $L \subset \Sigma^*$. This shows that L is countable.

Problem 2 Let L_1 be the language defined by the regular expression:

$$\boldsymbol{(a \cup ba)^*}$$

and let L_2 be the language defined by the regular expression:

$$\boldsymbol{(b \cup ba)^*}$$

(a) Let $S_1 = L_1 \cup L_2$. Write a regular expression that defines S_1. If such a regular expression does not exist, explain why.

Answer:

$$\boldsymbol{(a \cup ba)^* \cup (b \cup ba)^*}$$

(b) Complete the following sentence, using at most 4 words.

"The language L_1 consists of strings over the alphabet $\{a, b\}$ in which every b is . . . ".

Answer: The language L_1 consists of strings over the alphabet $\{a, b\}$ in which every b is followed by an a.

(c) Let $S_2 = L_1 \cap L_2$. Write five strings that belong to S_2. If such strings do not exist, explain why.

Answer: The language L_2 consists of strings over the alphabet $\{a, b\}$ in which every a is preceeded by a b.

The strings λ, *ba*, *baba*, *bababa*, *babababa* belong to S_2 because they meet both of our conditions.

(d) Write a regular expression that defines S_2. If such a regular expression does not exist, explain why. (Hint: Use your answer to (c) as a guide).

Answer:

$$\boldsymbol{(ba)^*}$$

(e) Let $S_3 = L_1 \setminus L_2$. Write a regular expression that defines S_3. If such a regular expression does not exist, explain why. (Hint: Think of some strings that belong to this language as a guide).

Answer: S_3 consists of strings where every b is followed by an a, but at least one a is not preceded by a b. These are strings are defined by the expression:

$$\boldsymbol{(a \cup ba)^* a (a \cup ba)^*}$$

Problem 3 Let L be the set of strings over alphabet $\{a, b\}$ that have odd length and have b for the middle character.

(a) Write a complete formal definition of a context-free grammar that generates L. If such a grammar does not exist, prove it.

Answer: $G = (V, \Sigma, P, S)$, where $\Sigma = \{a, b\}$, $V = \{S, Z\}$, and the production set P is:

$$\begin{aligned} S &\to ZSZ \mid b \\ Z &\to a \mid b \end{aligned}$$

(b) Write a regular expression that defines L. If such a regular expression does not exist, prove it.

Answer: Such an expression does not exist, since L is not a regular language. To prove this, assume the opposite. Let k be the constant as in the Pumping Lemma. Let $n > k$; then $a^n b a^n \in L$. In the "pumping" decomposition: $a^n b a^n = uvx$, we have: $|uv| \leq k < n$, hence the "pumping" substring v consists entirely of a's, say $v = a^j$. Recall that $j > 0$, since the "pumping" substring cannot be empty. By the pumping lemma, every word of the form $uv^i x$, $i \geq 0$, belongs to L. However, such a word has the form $a^{n+(i-1)j} b a^n$. If $i \neq 1$, the single b is not the middle character. Therefore, if $i \neq 1$, the pumped word does not belong to L, and this is a contradiction.

Problem 4 Let L be the set of strings over alphabet $\{a, b\}$ that have length at least 2 and have identical characters in the first and last positions.

(a) Write a complete formal definition of a context-free grammar that generates L. If such a grammar does not exist, prove it.

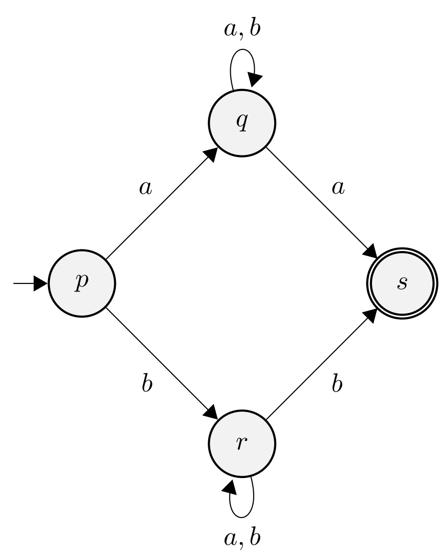
Answer:

$G = \{V, \Sigma, P, S\}$, where $\Sigma = \{a, b\}$,
$V = \{S, A\}$, and P is:

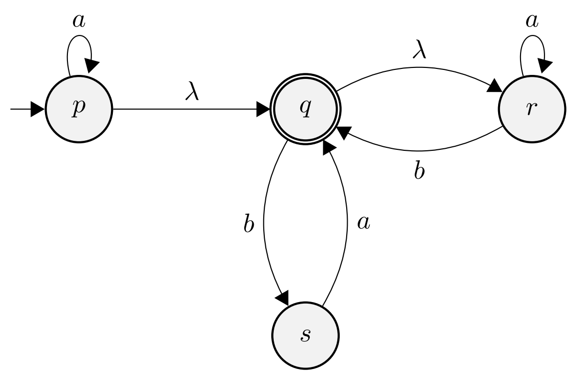
$$\begin{aligned} S &\to aAa \mid bAb \\ A &\to \lambda \mid aA \mid bA \end{aligned}$$

(b) Draw a state transition graph that represents a finite automaton that accepts L. If such an automaton does not exist, prove it.

Answer:

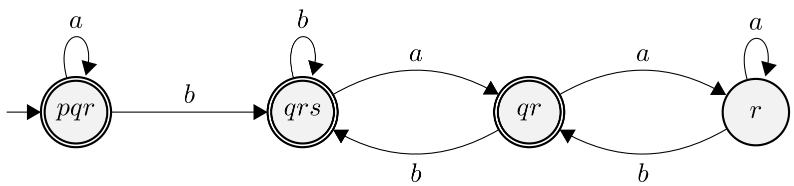


Problem 5 Let L be the language accepted by the NFA with the following state transition graph.

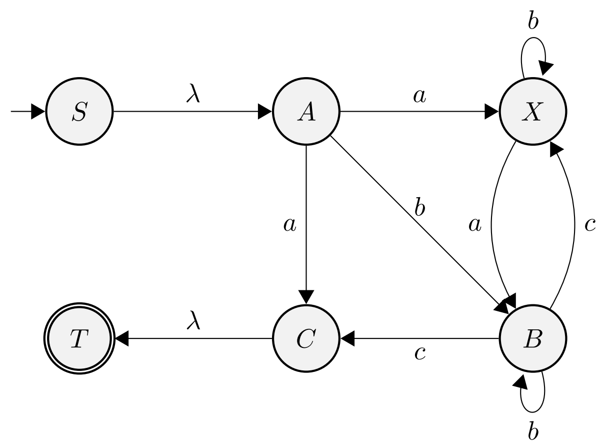


Draw a state-transition graph of a deterministic finite automaton that accepts L. If such an automaton does not exist, prove it.

Answer:



Problem 6 Consider the following finite automaton.



Draw the regular expression graph obtained from this automaton when the node X is eliminated using one step of the algorithm for conversion of a finite automaton to a regular expression. (Only show how to remove the node X. Do not complete the algorithm to obtain a regular expression that corresponds to the automaton.)

Answer:

There are two edges into node X (from nodes A and B). There is one edge out of X to B. Accordingly we must modify the labels on the edges from A to B and from B to B. The modifications add the expressions ab^*a and cb^*a, respectively. We obtain the following regular expression graph.