Practice problems on Euclidean Geometry and Euclidean Transformations.

Problem 1. Let ABC be a right triangle that is oriented clockwise and has angles of $90°$, $30°$, $60°$ at the vertices A, B, C.

(i) Identify $R_{C,120} \circ R_{B,60}$.

(ii) Identify $R_{C,-120} \circ R_{B,-60} \circ R_{A,-180}$.

Problem 2. Let $ABCDEF$ be a regular hexagon that is oriented clockwise (so that a rotation from A to B to C to D to E to F is clockwise).

(i) Identify $R_{D,120} \circ R_{A,60}$.

(ii) Identify $R_{F,180} \circ \rho_{ED} \circ R_{D,120}$.

Problem 3. Let $ABCDEFGH$ be a regular octagon that is oriented clockwise (so that a rotation from A to B to C to D to E to F to G to H is clockwise). Write O for the center of the octagon.

Identify the following transformations. For each state whether it is a translation, rotation, reflection, or glide reflection. Write down any point, angle or vector needed to identify the transformation. (For points or vectors that do not have names give a diagram to show how they are found.)

(i) $\rho_{GE} \circ \rho_{GA} \circ \rho_{HD}$

(ii) $\rho_{AE} \circ \rho_{BF}$

(iii) $R_{F,135} \circ R_{H,45}$

Problem 4. Let ABC be an equilateral triangle that is oriented clockwise (so that a rotation from A to B to C to A is clockwise).

In the following questions, identify any required lines, points, or vectors by giving either a name or a construction using the named points. For example, you might answer that a line is the perpendicular to BC from A.

(i) $\rho_{AB} \circ \rho_{BC}$ is a rotation. Identify its center and angle.

(ii) $\rho_{AB} \circ \rho_{BC} \circ \rho_{AB}$ is a reflection. Identify its mirror line.

(iii) $\rho_{AB} \circ \rho_{BC} \circ \rho_{CA}$ is a glide reflection γ_{XY}. Identify the vector XY.

Problem 5. Let ABC be a triangle with angles of $30°$, $60°$, and $90°$ at A, B, and C, respectively. Suppose that the triangle is oriented clockwise (so that a rotation from A to B to C to A is clockwise).

In the following questions, identify any required lines, points, or vectors by giving either a name or a construction using the named points. For example, you might answer that a line is the perpendicular to BC from A.

(i) $\rho_{AB} \circ \rho_{AC}$ is a rotation. Identify its center and angle.

(ii) $\rho_{AB} \circ \rho_{AC} \circ \rho_{AB}$ is a reflection. Identify its mirror line.

(iii) $\rho_{AB} \circ \rho_{BC} \circ \rho_{CA}$ is a glide reflection γ_{XY}. Identify the segment XY.

Problem 6. **CHOOSE ONE OPTION ONLY, there is no extra credit for doing both. Either:**

State and prove a theorem that describes the (Euclidean) transformations obtained by combining two reflections.

Or:

Let $A = (0,0)$, $B = (0,2)$, $C = (2,0)$. Let X and Y be the midpoints of the BC and CA. Identify the following 2 transformations (giving coordinates for any points or vectors that you use).

(i) $R_{A,90} \circ \tau_{BC}$

(ii) $\rho_{AB} \circ \rho_{AC} \circ \rho_{XY}$.

Answer:

Problem 7. **CHOOSE ONE OPTION ONLY, there is no extra credit for doing both. Either:**

State and prove a theorem that shows that (in Euclidean geometry) every isometry can be formed from at most three reflections.

Or:

Let ABC be an equilateral triangle that is oriented clockwise (so that a rotation from A to B to C to A is clockwise). For the following three transformations, identify any required lines, points, or vectors by giving either a name or a construction using the named points.

(i) $\rho_{AB} \circ \rho_{BC}$ is a rotation. Identify its center and angle.

(ii) $\rho_{AB} \circ \rho_{BC} \circ \rho_{AB}$ is a reflection. Identify its mirror line.

(iii) $\rho_{AB} \circ \rho_{BC} \circ \rho_{CA}$ is a glide reflection γ_{XY}. Identify the vector XY.

Answer:

Problem 8. Let $ABCD$ be a square with center O that is oriented counterclockwise (so that a rotation from A to B to C to D is counterclockwise). For the following three transformations, identify any required lines, points, or vectors by giving either a name or a construction using the named points.

(i) $R_A(90°) \circ R_B(90°)$ is a rotation. Identify its center and angle.

(ii) $\rho_{AD} \circ \rho_{AC} \circ \rho_{AB}$ is a reflection. Identify its mirror line.

(iii) $\rho_{BC} \circ \rho_{BD} \circ \rho_{AC}$ is a glide reflection γ_{XY}. Identify the points X and Y.

Answer:

Problem 9. Suppose that A, B, C, and D are the vertices $(0, 0)$, $(2, 0)$, $(2, 2)$, and $(0, 2)$ of a square.

(a) The transformation $R_{D,45} \circ R_{A,45}$ is a rotation by $90°$ about a point P. Find the coordinates of P.

(b) The transformation $\rho_{AC} \circ \gamma_{DA}$ can be decomposed as the combination of reflections across 4 mirror lines. Draw a diagram showing A, B, C, and D and the four mirror lines (marked in order by the numbers 1 to 4).

(c) Two of the mirrors in your answer to (b) can be moved so as to give a set of four mirror lines, three of which are parallel such that the combination of the corresponding reflections has the same result as that of (b). Draw a diagram showing these new mirror lines.

(d) The transformation $\rho_{AC} \circ \gamma_{DA}$ (as in (b) and (c)) simplifies to give a rotation by $90°$ about a center Q. Give coordinates for Q.