Practice problems on spherical trigonometry.

Problem 1. Find the missing sides and angles in each of the following cases for a spherical triangle ABC:

(a) $a = 60°$, $\beta = 90°$, $\gamma = 75°$.

(b) $\alpha = 65$, $\beta = 85$, $\gamma = 90$.

(c) $a = 90$, $b = 60$, $c = 100$.

(d) $\alpha = 85$, $b = 95$, $c = 105$.

Problem 2. In a spherical triangle ABC do the following properties hold?

(a) If $AB = AC$ are the base angles at B and C equal?

(b) If the angles at B and C are equal is it true that $AB = AC$?

(c) Do the angles add to $180°$?

(d) Do the sides add to $180°$?

(e) If $C = 90°$ is it true that $AB^2 = BC^2 + CA^2$?

(f) Do two triangles with equal corresponding sides have equal corresponding angles?

(g) Do two triangles with equal corresponding angles have equal corresponding sides?

Problem 3. Suppose that P is the north pole and points X and Y in the northern hemisphere are $45°$ apart and form a triangle PXY with angles $60°$ at X and $80°$ at P. Find the latitude of Y. Can you determine the longitude of Y?

Problem 4. Two points on the earth have latitude and longitude coordinates as follows: $A = (45°N, 60°W)$, $B = (60°N, 0°W)$. What direction should a plane fly to follow a great circle route from A to B? (Give your answer as the angle made to the direction of north at A.)

Problem 5. In a spherical triangle the angles at α, β and γ are $\pi/5$, $\pi/3$, $\pi/2$. Find the sum of the sides.

Problem 6. In a right angled spherical triangle $\alpha = a \neq 90°$. Find b and c.

Problem 7. In an equilateral spherical triangle show that $sec\alpha = 1 + sec a$.

Problem 8. Suppose that A, B, C and X are four points on the surface of a sphere. Such that:

(i) The point X lies on the geodesic from B to C.

(ii) The angles at A, B, and X of the spherical triangle ABX are $60°$, $60°$, and $90°$.

(iii) The geodesics AB and AC make an angle of $90°$.

Find the measures (in either degrees or radians) of the geodesics AB, AX, BX, CX, and AC and find the area of the spherical triangle ABC.

Answer:

Problem 9. Suppose that A, B, C and X are four points on the surface of a sphere. Such that:

(i) The point X lies on the geodesic from B to C (between B and C).

(ii) The angles at A, B, and X of the spherical triangle ABX are $60°$, $45°$, and $90°$.

(iii) The geodesics AB and AC make an angle of $90°$.

Find the measures (in either degrees or radians) of the geodesics AB, AX, BX, CX, and AC.

Answer: