Practice problems on hyperbolic geometry.

Problem 1 Prove that if P and Q are points of hyperbolic geometry then there is a unique hyperbolic line that passes through P and Q.

Problem 2 Define the hyperbolic distance between two h-points P and Q. Prove that if R is an h-point on the h-segment PQ then $d_h(P,Q) = d_h(P,R) + d_h(R,Q)$. (Here $d_h()$ denotes the hyperbolic distance function.)

Problem 3 Suppose that C is a cline that does not intersect the unit circle U. Prove that there is a Mobius transformation that transforms C and U to concentric circles. Deduce that there is a hyperbolic transformation that transforms C to a circle centered at the origin. Deduce that there exists an h-point Q and a constant d such that $C = \{P | d_h(P,Q) = d\}$.

Problem 4 Explain why SAS congruence applies in Hyperbolic geometry.

Problem 5 Let P be a point of the hyperbolic plane and let l be a hypebolic line that does not pass through P. Write ρ for the hyperbolic reflection in l, and let $Q = \rho(P)$. Let X be the point where l meets the hyperbolic line through P and Q.

(i) Draw a diagram to indicate the relationship of P, Q, X, and l considered as objects in the unit circle.

(ii) Prove that the hyperbolic line through P and Q is orthogonal to l. (You can use any convenient theorems of Mobius geometry, but you should give a clear statement of any theorem that you apply.)

(iii) Prove that $d_h(P,X) = d_h(X,Q)$. (Here d_h is the hyperbolic distance.)

Problem 6 Let P, Q, R be the points $(0,0)$, $(\frac{1}{2},\frac{1}{2})$, $(\frac{1}{2},0)$. Consider these three points as hyperbolic points in the unit disk model of the hyperbolic plane.

(a) Find the hyperbolic distances between P and Q and between P and R.

Answer:

(b) Find the hyperbolic angle between the hyperbolic lines joining P to Q and P to R.

Answer:

Problem 7 State and prove the isosceles triangle theorems of Hyperbolic geometry.

Problem 8 In hyperbolic geometry:

(a) Prove that if two h-lines are ultra parallel then they have a common perpendicular.

(b) How many common perpendiculars can be drawn to such a pair of h-lines? Explain your answer.

Problem 9 Suppose that P is a point of the hyperbolic plane and that l is a h-line of the hyperbolic plane that does not pass through P.

(a) How many h-lines that pass through P are perpendicular to l? Explain your answer.

(b) How many h-lines that pass through P are parallel to l? Explain your answer.

(c) How many h-lines that pass through P are ultra parallel to l? Explain your answer.