The following problems are exercises in hyperbolic trigonometry. You should use the following trigonometric formulas that we proved in class.

In a hyperbolic right angle triangle with hypotenuse c, legs a and b and angle A opposite the side a:

$$cosh(c) = cosh(a)cosh(b)$$

$$sin(A) = sinh(a)/sinh(c)$$

$$cos(A) = tanh(b)/tanh(c)$$

In a hyperbolic triangle with sides a, b, and c and angles A, B, C opposite these sides:

$$\frac{sinh(a)}{sin(A)} = \frac{sinh(b)}{sin(B)} = \frac{sinh(c)}{sin(C)}$$

$$cosh(c) = cosh(a)cosh(b) - sinh(a)sinh(b)cos(C)$$

$$cos(C) = -cos(A)cos(B) + sin(A)sin(B)cosh(c)$$

Problem 1 In a hyperbolic right angled triangle the two legs have hyperbolic lengths of 3 and 4. What is the hyperbolic length of the hypotenuse? Is this larger than, or smaller than 5?

Problem 2 A hyperbolic triangle has two sides with hyperbolic lengths 1 and 2. The included angle is $60°$. Find the length of the other side. Find the other two angles.

Problem 3 A hyperbolic quadrilateral has four equal sides and four angles of $60°$. Find the length of its sides. Is the quadrilateral cyclic?

Problem 4 Consider a regular hexagon that has angles of $90°$ (in hyperbolic geometry). Calculate the length of a side of the hexagon. (Hint: Does it help to consider the center of a circle that passes through the vertices of the hexagon?)

Problem 5 Suppose that an equilateral triangle (in hyperbolic geometry) has sides of length a. Determine $sinh(R)$, where R is the radius of a cicumscribed circle.