QUEENS COLLEGE — Department of Mathematics
Math 618 — Second Midterm Exam — Spring 2016 — 05.18.16
Solutions
6.00pm – 7.30pm, Wednesday, May 18, 2016

Problem 1. A hyperbolic triangle ABC has a right angle at C and sides with hyperbolic lengths a, b, c opposite angles A, B, $C = 90°$, respectively. The point X on the hypotenuse AB is the foot of a perpendicular from C. The hyperbolic length of CX is h.

Given that $\cosh(a) = 3$ and $\cosh(b) = 5$:

(i) Find $\sinh(a)$ and $\sinh(b)$. (These are both square roots of fractions, which you should give in their standard lowest terms.)

Answer:

$\sinh(a) = \sqrt{8}$ and $\sinh(b) = \sqrt{24}$.

(ii) Find $\tan(A)$. (This is the square root of a fraction, which you should give in its standard lowest terms.)

Answer:

$\tan(A) = \sqrt{\frac{1}{27}}$.

(iii) Find $\sinh(h)$. (This is the square root of a fraction, which you should give in its standard lowest terms.)

Answer:

$\sinh(h) = \sqrt{\frac{6}{7}}$.

Problem 2. Consider points $A = (1, 3)$ and $B = (2, 4)$ in the hyperbolic plane.

(i) Find the hyperbolic distance x between A and B. (This is the natural log of a fraction, Give the answer in these terms.)

Answer:

$\ln(\frac{3}{2})$

(ii) Find $\cosh(x)$.

Answer:

$\frac{13}{12}$

(iii) Find the angles between the hyperbolic line AB and the verticals at A and B. (Write your answers as inverse cosines.)

Answer:

$\cos^{-1}(\frac{-3}{5})$ and $\cos^{-1}(\frac{4}{5})$.

(iv) Find the hyperbolic area of the region above AB that lies between the vertical geodesics at A and B. (Leave your answers as a combination of two inverse trig functions.)

Answer:

$\cos^{-1}(\frac{3}{5}) - \cos^{-1}(\frac{4}{5})$.

Problem 3. What do we mean by a hyperbolic reflection across a curved geodesic?

Answer:

An inversion across the curved geodesic.

Prove the theorem that shows that a hyperbolic reflection is an isomtery.

Answer:

This is Theorem 4.4.1 on page 60 of the text.

Problem 4. Let ABC be the triangle with vertices $A = (0, 0)$, $B = (16, 12)$, $C = (25, 0)$. If l is a line we write ρ_l for the reflection across l.

(i) The combination $\rho_{AC} \circ \rho_{AB}$ is a rotation. Identify its center and the sine of its angle.

Answer:

The center is A and the angle has sine $\frac{-24}{25}$.

(ii) The combination $\rho_{AB} \circ \rho_{AC} \circ \rho_{BC}$ is a glide reflection $\gamma_{X,Y}$. Give coordinates for the points X and Y.

Answer:

The points X and Y could be $(16, 12)$ and $(16, -12)$. (Some other answers are also possible.)