QUEENS COLLEGE Department of Mathematics
Math 618 Final Exam Exam Spring 2015 05.20.15
Solutions
4pm – 6pm, Campbell Dome, Wednesday, May 20, 2015

Problem 1. Mark each as true or false:

(a) A combination of three glide reflections can never simplify to give a rotation.

F

(b) A spherical triangle with 3 equal sides need not have 3 equal angles.

F

(c) Inversion increases distances between points.

F

(d) For any x, we have: $\operatorname{csch}^2 x = 1 + \coth^2 x$.

F

(e) The hypotenuse of a right triangle can always be determined from the two legs in spherical geometry and in hyperbolic geometry.

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(f) In hyperbolic geometry a quadrilateral with four equal sides cannot have two of its angles as right angles.

F

Problem 2. Let $A = (0, 0)$, $B = (1, 0)$, $C = (1, 1)$, $D = (0, 1)$ be the vertices of a square.
Let X_1, X_2, X_3, X_4 be combinations of reflections given by:

$$X_1 = \rho_{CD} \circ \rho_{AC} \circ \rho_{BC}$$

$$X_2 = \rho_{AD} \circ \rho_{AB} \circ \rho_{BD}$$

$$X_3 = \rho_{CD} \circ \rho_{AC} \circ \rho_{DB} \circ \rho_{AB}$$

$$X_4 = \rho_{DA} \circ \rho_{CD} \circ \rho_{BC} \circ \rho_{AB}$$

Which if any of X_1, X_2, X_3, X_4 are:

(a) The identity?

None

(b) A reflection? For each of the transformations that is a reflection give the equation for a mirror line.

X_1**, mirror line** $y = x$**.**

(c) A rotation? For each of the transformations that is a rotation give coordinates for the center and the directed angle.

X_3**, center** $(.5, .5)$ **angle** $180°$

(d) A translation? For each of the transformations that is a translation give coordinates for its vector.

X_4**, vector** $(-2, 2)$**.**

(e) A glide reflection? For each of the transformations that is a glide reflection give the equations for three mirrors (two parallel and one perpendicular) that define it.

X_2**, mirrors** $y = 1 - x$**,** $y = -x$**,** $y = x$**.**

(f) A dilation? For each of the transformations that is a dilation give coordinates for the center and the scale factor.

None

Problem 3. Two points on the earth have latitude and longitude coordinates as follows: $A = (45°N, 45°W)$, $B = (30°N, 45°E)$.

(a) A plane is to fly the great circle route from A to B. Let α be the angle made from its path to the direction of north at A. Find $\tan \alpha$.

$\tan\alpha = \sqrt{\frac{2}{3}}$.

(b) Another plane is to fly the great circle route from B to A. Let β be the angle made from its path to the direction of north at B. Find $\tan\beta$.

$\tan\beta = 2$.

Problem 4. A hyperbolic triangle ABC has a right angle at A and angles of $30°$ at B and $45°$ at C. Find the values of $\cosh(a)$, $\sinh(a)$, $\cosh(b)$, $\sinh(b)$, $\cosh(c)$ and $\sinh(c)$.

Answer:

$\cosh(a) = \sqrt{3}$, $\sinh(a) = \sqrt{2}$, $\cosh(b) = \sqrt{\frac{3}{2}}$, $\sinh(b) = \sqrt{\frac{1}{2}}$, $\cosh(c) = \sqrt{2}$ **and** $\sinh(c) = 1$.

Problem 5. State and prove a theorem about the relationship between orthogonal circles and the effect of inversion of one of the circles across the other.

Problem 6. A hyperbolic isosceles triangle has sides with lengths $a = \cosh^{-1} 3$, $b = \cosh^{-1}\sqrt{5}$, $c = \cosh^{-1}\sqrt{5}$. Find the angles of the triangle. (You can leave your answers in terms of inverse trig functions, unless they happen to simplify to give a standard angle.)

Answer:

$60°$, $\cos^{-1}\sqrt{\frac{5}{8}}$, $\cos^{-1}\sqrt{\frac{5}{8}}$.

Problem 7. Find the area of the quadrilateral in the hyperbolic plane with vertices $A = (0,1)$, $B = (1,2)$, $C = (1,3)$, $D = (0,4)$. (Leave your answer as a sum or difference of inverse trig functions.)

Answer:

$\sin^{-1}\frac{2}{\sqrt{5}} - \sin^{-1}\frac{1}{\sqrt{5}} - \sin^{-1}\frac{4}{5} + \sin^{-1}\frac{3}{5}$