QUEENS COLLEGE | Department of Mathematics
Math 618 | Second Midterm Exam Spring 2015 05.13.15

Solutions

6.00pm – 7.30pm, Wednesday, May 13, 2015

Problem 1. *(10 points)* State and prove a theorem that shows that (in Euclidean geometry) every isometry can be formed as a combination of at most three reflections.

Problem 2. *(10 points)* Let ABC be the triangle with vertices $A = (0, 0)$, $B = (1, \sqrt{3})$, $C = (0, \sqrt{3})$.

(i) The combination $\rho_{AB} \circ \rho_{AC}$ is a rotation. Identify its center and angle.

Answer:

Center: A

angle: $-60°$.

(ii) The combination $\rho_{AB} \circ \rho_{AC} \circ \rho_{BC}$ is a glide reflection $\gamma_{X,Y}$. Give coordinates for the points X and Y.

Answer:

$X = (0, \sqrt{3}), Y = (\frac{3}{2}, \frac{\sqrt{3}}{2})$.

Problem 3. *(10 points)* A hyperbolic triangle ABC has sides with lengths a, b, c opposite angles α, β, γ, respectively. Given that $\sinh(a) = 1$, $\cosh(c) = \sqrt{3}$, $\beta = 30°$.

(i) Find $\cosh(a)$ and $\sinh(c)$. (You must simplify these answers to get full credit!)

Answer:

$\cosh a = \sqrt{2}$, $\sinh c = \sqrt{2}$.

(ii) Find $\cosh(b)$ and $\sinh(b)$. (You must simplify these answers to get full credit!)

Answer: $\cosh(b) = \sqrt{\frac{3}{2}}$, $\sinh(b) = \sqrt{\frac{1}{2}}$.

(iii) Find $\sin(\gamma)$. (You must simplify this answer to get full credit!)

Answer:

$\sin(\gamma) = 1$.

Problem 4. *(10 points)* A triangle ABC has a right angle at A and an angle of $45°$ at B. Give formulas for the length of AC and the angle at C in the following three situations.

(i) The geometry is Euclidean and $AB = \frac{1}{2}$.

Answer:

$C = 45°$, $b = \frac{1}{2}$.

(ii) The geometry is spherical and $\sin(AB) = \frac{1}{2}$.

Answer:

$C = \cos^{-1}(\frac{\sqrt{3}}{\sqrt{8}})$, $b = \tan^{-1}\frac{1}{2}$.

(iii) The geometry is hyperbolic and $\sinh(AB) = \frac{1}{2}$.

Answer:

$C = \cos^{-1}(\frac{\sqrt{5}}{\sqrt{8}})$, $b = \tanh^{-1}\frac{1}{2}$.

Problem 5. *(10 points)* Consider the points $A = (0, 2)$, $B = (0, 4)$ and $C = (1, 3)$ as points in the hyperbolic plane. Let γ be the geodesic from A to B and let δ be the geodesic from B to C.

(i) Calculate the hyperbolic length of γ. (Leave your answer in terms of the function ln but simplify all other terms.)

Answer:

$\ln 2$

(ii) Calculate the hyperbolic length of δ. (Leave your answer in terms of the function ln but simplify all other terms.)

Answer:

$\ln\frac{3}{2}$

(iii) Calculate the hyperbolic angle of triangle ABC at B. (This is an angle between γ and δ.) Leave your answer in terms of the function cos^{-1} but simplify all other terms.

Answer:

$\cos^{-1}\frac{3}{5}$