QUEENS COLLEGE Department of Mathematics
Math 618 Second Midterm (Early Alternate) Exam Spring 2015 05.11.15
Solutions
5pm – 6.30pm, Monday, May 11, 2015

Problem 1. *(10 points)* Prove the theorem which states that an inversion whose center lies on the x-axis is a hyperbolic isometry.

Problem 2. *(10 points)* Let ABC be the triangle with vertices $A = (0, 0)$, $B = (\sqrt{3}, 1)$, $C = (\sqrt{3}, 0)$.
(i) The combination $\rho_{AB} \circ \rho_{AC}$ is a rotation. Identify its center and angle.
Answer:
Center: A
angle: $60°$.
(ii) The combination $\rho_{AB} \circ \rho_{AC} \circ \rho_{BC}$ is a glide reflection $\gamma_{X,Y}$. Give coordinates for the points X and Y.
Answer:
$X = (\sqrt{3}, 0), Y = (\frac{\sqrt{3}}{2}, \frac{3}{2})$.

Problem 3. *(10 points)* A hyperbolic triangle ABC has sides with lengths a, b, c opposite angles α, β, γ, respectively. Given that $\sinh(a) = 1$, $\cosh(c) = \sqrt{3}$, $\beta = 45°$.
(i) Find $\cosh(a)$ and $\sinh(c)$. (You must simplify these answers to get full credit!)
Answer:
$\cosh a = \sqrt{2}$, $\sinh c = \sqrt{2}$.
(ii) Find $\cosh(b)$ and $\sinh(b)$. (You must simplify these answers to get full credit!)
Answer:
$\cosh(b) = \sqrt{6} - 1$, $\sinh(b) = \sqrt{7 - 2\sqrt{6}}$.
(iii) Find $\sin(\gamma)$. (You must simplify this answer to get full credit!)
Answer:
$\sin(\gamma) = \frac{1}{\sqrt{7-2\sqrt{6}}}$.

Problem 4. *(10 points)* A triangle ABC has a right angle at A and an angle of $30°$ at B. Give formulas for the length of AC and the angle at C in the following three situations.
(i) The geometry is Euclidean and $AB = \frac{1}{\sqrt{2}}$.
Answer:
$C = 60°$, $b = \frac{1}{\sqrt{6}}$.
(ii) The geometry is spherical and $AB = 45°$.
Answer:
$C = \cos^{-1}(\frac{1}{\sqrt{8}})$, $b = \tan^{-1}\frac{1}{\sqrt{6}}$.
(iii) The geometry is hyperbolic and $\sinh AB = \frac{1}{\sqrt{2}}$.
Answer:
$C = \cos^{-1}(\frac{\sqrt{3}}{\sqrt{8}})$, $b = \tanh^{-1}\frac{1}{\sqrt{6}}$.

Problem 5. *(10 points)* Consider the points $A = (0, 2)$, $B = (0, 4)$ and $C = (1, 3)$ as points in the hyperbolic plane. Let γ be the geodesic from A to B and let δ be the geodesic from A to C.
(i) Calculate the hyperbolic length of γ. (Leave your answer in terms of the function *ln* but simplify all other terms.)
Answer:
$\ln 2$
(ii) Calculate the hyperbolic length of δ. (Leave your answer in terms of the function *ln* but simplify all other terms.)
Answer:
$\ln\frac{3(\sqrt{13}+3)}{2(\sqrt{13}+2)}$
(iii) Calculate the angle of the hyperbolic triangle ABC at A. This is between δ and γ. (Leave your answer in terms of the function cos^{-1} but simplify all other terms.)
Answer:
$\cos^{-1}\frac{3}{\sqrt{13}}$