QUEENS COLLEGE Department of Mathematics
Math 618 Final Exam Exam Spring 2010 05.24.10
Instructor: Alex Ryba
6.15pm – 8.15pm, Monday, May 24, 2010
Complete all of the following information.

STUDENT LAST NAME (PRINT): ______________________________

STUDENT First NAME (PRINT): ______________________________

LAST 4 digits of student ID#: ______________

SIGNATURE: ______________________________

Useful formulas:

$$\sinh x = \frac{e^x - e^{-x}}{2}, \qquad \cosh x = \frac{e^x + e^{-x}}{2}$$

$$\text{Hyperbolic law of sines } \frac{\sinh a}{\sin \alpha} = \frac{\sinh b}{\sin \beta} = \frac{\sinh c}{\sin \gamma}$$

$$\text{Hyperbolic laws of cosines } \cosh b \cosh c - \cosh a = \sinh b \sinh c \cos \alpha, \qquad \cos \beta \cos \gamma + \cos \alpha = \sin \beta \sin \gamma \cosh a$$

$$\text{Spherical law of sines } \frac{\sin a}{\sin \alpha} = \frac{\sin b}{\sin \beta} = \frac{\sin c}{\sin \gamma}$$

$$\text{Spherical laws of cosines } \cos b \cos c - \cos a = -\sin b \sin c \cos \alpha, \qquad \cos \beta \cos \gamma + \cos \alpha = \sin \beta \sin \gamma \cos a$$

THIS IS A CLOSED BOOK TEST. NO BOOKS, NOTES, COMPUTERS, OR CELL PHONES ARE ALLOWED.

A CALCULATOR IS PERMITTED.

It is department policy to give a **grade of F** to any student **who uses prohibited material** or helps or receives help from any other student during an exam.

The exam has **7** problems, you should answer exactly **5** of them. If you answer more than 5 questions, I will grade only the first 5 that I find. Badly explained and unreadable answers will receive little credit.

If your printed exam is missing any problem please notify the proctor as soon as possible.

Answer the problems in the spaces provided.

problem:	01	02	03	04	05	06	07	Σ
grade:								

 # 1. *-S10-618-Fin6*

Problem 1. *(6 points)* **Either:**

State and prove a theorem that describes the equivalence of Euclid's 5th postulate, Playfair's postulate, and the fact that the angle sum of a triangle is $180°$.

Or:

Let $ABCDEF$ be a regular hexagon that is oriented clockwise (so that a rotation from A to B to C to D to E to F is clockwise). Identify $R_{D,60} \circ \rho_{ED} \circ R_{D,120}$.

Choose one option only.

Answer:

Problem 2. *(6 points)* **Either:**

State and prove the AAA congruence theorem of hyperbolic geometry.

Or:

Consider the points $A = (0, 1)$, $B = (6, 1)$ and $C = (12, 1)$ that form a triangle in the hyperbolic plane. Find the angles and area of this triangle.

Choose one option only.

Answer:

Problem 3. *(6 points)* Let I be an inversion and let C be a circle such that $I(C)$ is also a circle. When do C and $I(C)$ have equal radii? (Explain your answer.)

Problem 4. *(6 points)* Suppose that A, B, C and X are four points on the surface of a sphere. Such that:

(i) The point X lies on the geodesic from B to C.

(ii) The angles at A, B, and X of the spherical triangle ABX are $60°$, $60°$, and $90°$.

(iii) The geodesics AB and AC make an angle of $90°$.

Find the measures (in either degrees or radians) of the geodesics AB, AX, BX, CX, and AC and find the area of the spherical triangle ABC.

Answer:

Problem 5. *(6 points)* Suppose that a hyperbolic hexagon $ABCDEF$ is inscribed in a hyperbolic circle, that its sides have equal hyperbolic lengths, and that its angles all have a measure of $60°$. Calculate the hyperbolic length of the side AB.

(Hint: Let O be the center of the circle and consider triangle OAB.)

Answer:

Problem 6. *(6 points)* Let X and Y be disjoint circles such that X lies inside Y. Suppose that circles C_1, C_2, C_3, C_4, and C_5 are arranged so that each touches both the circles X and Y, so each circle C_i touches the circle C_{i+1} when $1 \leq i \leq 4$ and so that C_5 touches C_1.

(i) Draw a diagram that shows the relationship of the 7 circles.

(ii) What sort of transformation can be applied to transform two of the circles to concentric circles?

(iii) Prove that there is an eighth circle that is orthogonal to each circle C_i.

Problem 7. *(6 points)* Consider the following claims as statements of hyperbolic geometry (using the upper half plane model). Mark each as true or false:

(a) If two hyperbolic triangles have the same area then they have the same angles.

(b) Any arc of a Euclidean circle in the upper half plane is either an arc of a hyperbolic circle or a segment of a hyperbolic straight line.

(c) The (hyperbolic) distance between the points $(0, 1)$ and $(2, 3)$ equals the (hyperbolic) distance between the points $(0, 3)$ and $(6, 9)$.

(d) The exterior angle of a (hyperbolic) triangle is less than the sum of the two opposite interior angles.

(e) Hyperbolically congruent figures have equal (hyperbolic) areas.

(f) If $\mathcal{C}$ is the arc of a circle joining $(0, 1)$ to $(5, 10)$ then the (hyperbolic) length of $\mathcal{C}$ is less than the length of any other curve joining these points.