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These problems were given on exams for this course. Some older problems did not make use of generics in Java, but generic implementations are now required in this course.

**Problem 1** A generic priority queue is implemented as a heap so that  $n$  entries of comparable type  $K$  occupy elements  $1, 2, 3, \dots, (n + 1)$  of an array *data* in the heap. Usual heap order and heap shape requirements are in force. (Note this uses slightly different array elements from the implementation described in class and in the textbook.) A skeleton for the class is as follows:

```
public class HeapPriorityQueue // class title line to be completed as (a)
{ private K data[]; private int size = 0; private int capacity = 100;
  // constructor to be coded as (b)
  public void insert(K x) throws Exception {
    if (size >= capacity - 2) throw new Exception("Priority Queue Full");
    data[++size] = x;
    bubbleUp(size);
  }
  public K removeMin() throws Exception { // omitted
  private void swapData(int n, int m) { // omitted, swaps entries n and m
  private void bubbleUp(int n) { // omitted to be coded as (c)
  private void bubbleDown(int n) { // omitted
}
```

- (a) Write the complete class title line, including a clause that makes it implement a *PriorityQueue*.
- (b) Implement a constructor with no arguments.
- (c) Implement the method `bubbleUp`.

**Problem 2** The standard interface `PriorityQueue` and class `HeapPriorityQueue` include the following code.

```
interface PriorityQueue<K extends Comparable<K>> {
  public void add(K x) throws Exception;
  public K removeMin() throws Exception;
}

class HeapPriorityQueue<K extends Comparable<K>> implements
  PriorityQueue<K> {
  private K data[]; // the root is stored at index 0 in the array
  private int size;
  private int capacity;
  // constructors, add and removeMin method code omitted
  /* methods with titles
  void bubbleUp(int n)
  void bubbleDown(int n)
  void swapData(int n, int m)
  are available, but the code is not shown here */
}
```

Write complete code for a non-standard `HeapPriorityQueue` method `removeSecondMin` that efficiently removes and returns the second minimum element from the data structure. Your solution can make use of the methods `bubbleUp`, `bubbleDown` and `swapData` but must not apply the constructor or either of the methods `add` and `removeMin`.

For example, if the array contains the following elements:

1, 2, 3, 4, 5,

It should be changed by `removeSecondMin` to

1, 4, 3, 5,

Inefficient and excessively complicated solutions will lose points.

**Answer:**

**Problem 3** (a) What two properties of a binary tree make it a heap?

(b) Give a Java implementation method for the method:

```
public static <K extends Comparable<K>> boolean hasHeapOrder(BNode<K> r)
```

Here  $r$  is a node in a Binary Tree, so that  $r$  has instance variables *data*, *parent*, *left* and *right*. The method should return true if the subtree rooted at  $r$  satisfies the heap ordering requirement.

**Problem 4** (a) What two properties of a binary tree make it a heap?

(b) Give either a pseudocode outline (or for extra credit, a Java method) for an algorithm:

```
public static <K> boolean hasHeapShape(BNode<K> r)
```

That returns true, if the subtree rooted at  $r$  satisfies the heap shape requirement.