These problems were given on exams for this course. Some older problems did not make use of generics in Java, but generic implementations are now required in this course.

**Problem 1**  A generic priority queue is implemented as a heap so that $n$ entries of comparable type K occupy elements 1, 2, 3, ...(n + 1) of an array data in the heap. Usual heap order and heap shape requirements are in force. (Note this uses slightly different array elements from the implementation described in class and in the textbook.) A skeleton for the class is as follows:

```java
public class HeapPriorityQueue // class title line to be completed as (a) {
    private K data[]; private int size = 0; private int capacity = 100;
    // constructor to be coded as (b)
    public void insert(K x) throws Exception {
        if (size >= capacity - 2) throw new Exception("Priority Queue Full");
        data[++size] = x;
        bubbleUp(size);
    }
    public K removeMin() throws Exception { // omitted
        private void swapData(int n, int m) { // omitted, swaps entries n and m
            private void bubbleUp(int n) { // omitted to be coded as (c)
                private void bubbleDown(int n) { // omitted
            }
}
```

(a) Write the complete class title line, including a clause that makes it implement a `PriorityQueue`.
(b) Implement a constructor with no arguments.
(c) Implement the method `bubbleUp`.

**Problem 2**  (a) What two properties of a binary tree make it a heap?

(b) Give a Java implementation method for the method:

```java
public static <K extends Comparable<K>> boolean hasHeapOrder(BNode<K> r)
```

Here `r` is a node in a Binary Tree, so that `r` has instance variables `data, parent, left` and `right`. The method should return true if the subtree rooted at `r` satisfies the heap ordering requirement.

**Problem 3**  (a) What two properties of a binary tree make it a heap?

(b) Give either a pseudocode outline (or for extra credit, a Java method) for an algorithm:

```java
public static <K> boolean hasHeapShape(BNode<K> r)
```

That returns true, if the subtree rooted at `r` satisfies the heap shape requirement.