Problem 1  Give useful \( \Theta \) estimates for the following functions \( t(n) \).

(a) \( t(n) = 5\log_2(n^2) + (\log_2(n))^2 + \log_4(n) + (\log_2(100))^3 \).

(b) \( t(n) \) satisfies \( t(n) = 2t(n/2) + n \).

(c) \( t(n) \) satisfies \( t(n) = 4t(n/3) + n \).

(d) \( t(n) \) is the running time of the following function:

```java
public static void shuffle(int []x, int a, int b, int n) {
    for (int i = 0; i < n; i+=2) {
        int temp = x[a + i];
        x[a + i] = x[b + i];
        x[b + i] = temp;
    }
}
```

(e) \( t(n) \) is the running time of the following function that calls shuffle from (d):

```java
public static void multiShuffle(int []x, int a, int n) {
    if (n == 0) return;
    multiShuffle(x, a, n/2);
    multiShuffle(x, a + n/4, n/2);
    multiShuffle(x, a + n/2, n/2);
    shuffle(x, a, a + n/2, n/2);
}
```

Problem 2  Give useful \( O \)-estimates of the run times of the following methods:

(a) The method `addHead` for a singly linked list that has size \( n \).

(b) An efficient method to calculate the power \( x^n \) (consider the run time as a function of \( n \), the time should be considered as being proportional to the total number of additions, subtractions, multiplications, and divisions performed).

(c) An efficient method to sort an array of \( n \) numbers into order.

For (d) and (e), consider the following recursive function, in which \( A \) represents an integer constant:

```java
int f(int n) {
    if (n <= 0) return 1;
    int ans = f(n/2) * 2;
    for (int i = 1; i<= n; i++)
        for (int j = 1; j <= n; j++)
            ans += i / j;
    for (int k = 1; k < A; k++)
        ans -= f(n/2 - k);
    return ans;
}
```

(d) In the case where \( A = 3 \) estimate the run time of \( f(n) \).

(e) In the case where \( A = 4 \) estimate the run time of \( f(n) \).
Problem 3  Give useful O estimates for the run times of the following methods.

(a) removeMin for a PriorityQueue storing n items in a heap implementation.

(b) preOrder for a general Tree storing n items.

(c) get for a chained HashTable storing n items with load factor λ.

(d) A recursive method f that processes n input items by: sorting the items (efficiently), makes two recursive calls to process n/2 items, computes the products of all pairs of input items and finally makes two further recursive calls to process n/2 items.