Problem 1  (10 points)  Give useful Θ estimates for the following functions \( t(n) \).

(a) \( t(n) = \log_2(10^n) + 10\sqrt{n} + \log_2(n^{10}) + (\log_2(n))^2 \).

**Answer:** \( \Theta(n) \)

(b) \( t(n) \) is the running time for the `postOrder` method applied to a binary tree that contains \( n \) nodes.

**Answer:** \( \Theta(n) \)

(c) \( t(n) \) satisfies \( t(n) = 6t(n/2) + (n - 1)n(n + 1)/6 \).

**Answer:** \( \Theta(n^3) \)

Consider the following binary tree:

```
        Z
       / \
      /   \
     X    C
    / \   / \
   V B N M
  / \ / \ / \ / \
 A S D F G H
 / \
J
```

(d) What is the height of the node marked \( X \) in the illustrated binary tree? **Answer:** 2

(e) What is the depth of the node marked \( X \) in the illustrated binary tree? **Answer:** 1

Problem 2  (10 points)  Consider the following code:

```java
import java.util.Stack;

public class Problem1 {
    public static void main(String args[]) {
        Stack<Integer> s = setStack(5); printStack("A", s);  // line A
        s = setStack(6); stackStack(s); printStack("B", s);  // line B
        s = setStack(8); cutStack(s); printStack("C", s);  // line C
        s = setStack(8); s = cutStack(s); printStack("D", s);  // line D
        s = setStack(7); s = cutStack(s); printStack("E", s);  // line E
    }

    public static Stack<Integer> setStack(int n) {
        Stack<Integer> ans = new Stack<>();
        for (int i = 1; i <= n; i++) ans.push(i);
        return ans;
    }

    public static void printStack(String tag, Stack<Integer> s) {
        System.out.print(tag + " ");
        while (!s.empty()) System.out.print(s.pop());
        System.out.println();
    }

    public static Stack<Integer> cutStack(Stack<Integer> s) {
        Stack<Integer> ans = new Stack<>();
        while (!s.empty()) {
            ans.push(s.pop()); s.pop();
```
public static void stackStack(Stack<Integer> s) {
    if (s.empty()) return;
    int x = s.pop();
    stackStack(s);
    s.push(x);
}

(a) What is the output from line A?
Answer: A 54321
(b) What is the output from line B?
Answer: B 654321
(c) What is the output from line C?
Answer: C
(d) What is the output from line D?
Answer: D 2468
(e) What is the output from line E?
Answer: No output. An Exception is thrown and printed.

Problem 3  (10 points) The class BNode implements a generic binary tree node. The class provides the following instance variables and methods.

public class BNode<T> {
    BNode<T> parent, left, right;
    T data;
    // a constructor and getters and setters for instance variables, code omitted
    public int lowerLeaves() // to be written
}

Write complete code for the method lowerLeaves that counts the leaves in the subtree beginning at the node.

For example, if lowerLeaves() is called at the node storing X in the following tree, the result is 2 corresponding to the nodes storing M and A. (Note that M and A appear in the tree underneath X. The leaf N does not and is not counted.)

```
Z
 / \
X   C
 / \  
V   B  N
 / \  
M   A
```

Code longer than 10 lines may be subject to a penalty. Code with a running time worse than \(O(n)\) (where \(n\) is the number of nodes in the subtree being considered) will be subject to a penalty.

Answer:

```java
public int lowerLeaves() {
    if (left == null && right == null) return 1;
    if (left == null) return right.lowerLeaves();
    if (right == null) return left.lowerLeaves();
    return left.lowerLeaves() + right.lowerLeaves();
}
```